Proposal for an Easier Obtention of Piezoresistance Coefficients

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Abstract

An alternative procedure to obtain the piezoresistance coefficients is proposed and implemented. It consists on the integration of the piezoresistance equation that relates the resistance change with the pressure change, via the piezoresistance coefficients. This yields a linear dependence between the logarithm of the measured resistance (R) and pressure (P). When implemented experimentally, this facilitates to make one single measurement of R vs P. The linear fit of the logarithm plot yields the piezoresistance coefficients. This technique is compared with the usual procedure, yielding smaller errors, while saving time and experimental and calculation effort, as only one measurement is necessary, as compared with the former procedure.

Keywords: Piezoresistance, Piezoresistance coefficients, Hydrostatic pressure.

The piezoresistance effect consists on the change of electrical resistance as function of pressure. In particular, in the semiconductor industry, it has important applications in Micro Electro Mechanical Systems (MEMS), such as sensors, accelerometers and solid-state joysticks [1].

These MEMS devices base their functionality on the fact, that in the case for semiconductors, the change of the resistivity tensor in a crystal can be written as a tensor equation related to the stress tensor as

\[ \frac{\Delta \rho}{\rho_0} = \pi \sigma \]

where \( \Delta \rho \), \( \pi \) and \( \sigma \) are the change of resistivity tensor, piezoresistance tensor and the stress tensor, respectively. Each one as dimensions of \( 3\times3 \), \( 3\times3\times3\times3 \) and \( 3\times3 \), respectively while \( \rho_0 \) is the initial resistivity. Using crystal properties and symmetries, the notation can be collapsed and a simpler equation is obtained:

\[ \begin{pmatrix} \frac{\Delta \rho_1}{\rho_0} \\ \frac{\Delta \rho_2}{\rho_0} \\ \frac{\Delta \rho_3}{\rho_0} \\ \frac{\Delta \rho_4}{\rho_0} \\ \frac{\Delta \rho_5}{\rho_0} \\ \frac{\Delta \rho_6}{\rho_0} \end{pmatrix} = \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} \]

Further details about notation collapse and the use of the crystal symmetries can be found in Ref. 1, as they are not the intention of this article.

In particular, when stress is hydrostatic pressure, i.e., homogeneous and isotropic, then \( \sigma_1 = \sigma_2 = \sigma_3 = -\Delta P \) and \( \sigma_4 = \sigma_5 = \sigma_6 = 0 \), and Eq. 3 becomes

\[ \begin{pmatrix} \frac{\Delta \rho_1}{\rho_0} \\ \frac{\Delta \rho_2}{\rho_0} \\ \frac{\Delta \rho_3}{\rho_0} \\ \frac{\Delta \rho_4}{\rho_0} \\ \frac{\Delta \rho_5}{\rho_0} \\ \frac{\Delta \rho_6}{\rho_0} \end{pmatrix} = \begin{pmatrix} \pi_{11} + 2\pi_{12} \\ \pi_{11} + 2\pi_{12} \\ \pi_{11} + 2\pi_{12} \\ 0 \\ 0 \\ 0 \end{pmatrix} (-\Delta P) \]

Which becomes one single linear equation

\[ \frac{\Delta \rho}{\rho} = \frac{\Delta R}{R} = -\left( \pi_{11} + 2\pi_{12} \right) \Delta P \]

where it has been used the fact that the resistivity is proportional to the measured resistance R. Also notice the minus sign that has been introduced in \( \sigma_1 = \sigma_2 = \sigma_3 = -\Delta P \). This is due to the nature of hydrostatic pressure, which is a comprehensive stress, and thus, has a negative sign.

Hence, a piezoresistance measurement done with hydrostatic...
pressure is a useful technique to obtain information about the piezoresistance coefficients $\pi_{11}$ and $\pi_{12}$.

The standard procedure found in the literature is to measure the resistance $R_i$ at atmosphere pressure $P_0$, then increase pressure to different values of $P_1$, $P_2$, $P_3$, … while measuring the respective $R_1$, $R_2$, $R_3$, … The respective $\Delta R_i = R_i - R_{i-1}$ and $\Delta P_i = P_i - P_{i-1}$ are obtained for each $i = 1, 2, 3$ measurement, and using Eq. 4, the value of $-(\pi_{11} + 2\pi_{12})$ is obtained [1 -13].

This procedure has the inconvenience that the pressure should be set at certain value $P_i$, the resistance $R_i$ measured at this value, and the respective $\Delta R = R_i - R_{i-1}$ and $\Delta P_i = P_i - P_{i-1}$ be calculated in order to use Eq. 4. A more convenient way to proceed would be a continuous change on the set pressure $P$ while measuring continuously the resistance $R$. This second way to proceed would yield a more complete set of data, providing more information. It would be also very convenient if at the same time it could possible to avoid the calculation of $\Delta R_i$ and $\Delta P_i$ to obtain the piezoresistance coefficient, in order to save time and calculation efforts.

On what follows an alternative procedure to achieve this is proposed.

If the limit $\Delta P \rightarrow 0$ is done on Eq. 4, one obtains:

$$\frac{1}{R} \frac{dR}{dP} = \frac{d(lnR)}{dP} = -(\pi_{11} + 2\pi_{12})$$

(5)

Integrating Eq. 5 respect to $P$, yields

$$lnR = -(\pi_{11} + 2\pi_{12})P + lnR_0$$

(6)

Where $R_0$ is the resistance at atmosphere pressure.

Eq. 6 suggests that a continuous piezoresistance measurement can be done, increasing $P$ from atmosphere pressure to larger values, while constantly measuring $R$. Once the measurement is done, the data are plot as $ln R$ vs $P$ and the slope of the linear fit yields $-(\pi_{11} + 2\pi_{12})$.

In order to implement this idea, a monocrystal of n-type <100>Si is used, as Si has been extensively studied [14,15]. Indium (In) was always used for ohmic contacts. Before any In soldering, the surface was slightly scratch with a knife to remove any native oxide. First, four soldered In ohmic contacts were done on the edge of a square sample, i.e, in van der Pauw geometry, such as Figure 1[16,17]. A Hall measurement was done and a resistivity of 110 m$\Omega$·cm and an n-type charge carrier density of $n=8\times10^{16}$ cm$^{-3}$ was obtained. This is in agreement with the reported electrical values of Si [18,19]. Afterwards a 2 mm $\times$ 4 mm slide was cut and four In soldered ohmic contacts where place on it, in the four probe configuration. Our sample was introduced in our Berilium high pressure cell, provided by the Institute of High Pressure Physics (IHPP, Warsaw, Poland). As recommended in their manual, a current of 10 µA was introduced on the edge In ohmic contacts, while measuring the voltage drop in the inner ohmic contacts, to measure the sample resistance. Pressure was varied from atmosphere pressure up to 1260 MPa. Measurements were done continuously, making time stops at pressures around 97, 122, 141, 157, 182, 204, 231, 258, 286, 318, 351, 388, 426, 467, 509, 557, 611, 661, 714, 770, 831, 890, 973, 1039, 1105, 1182 and 1260 MPa, as can be shown in Figure 1. This was done in order to obtain $R_i$ resistance values at definite $P_i$ pressures, to compare with the usual procedure that is done in the literature, as it is shown below.

In Figure 1 the measurement and its linear fitting are shown.

![Figure 1: Resistance vs Pressure measurement. The linear fit is shown in red, yielding a slope of $2.06\times10^{-5}$ (MPa)$^{-1}$. The original measurement is shown in black](image)

The linear fit yields a slope of $2.06\times10^{-5}$ (MPa)$^{-1}$ with a standard error in slope of $1.171\times10^{-7}$ (MPa)$^{-1}$.

The average $R_i$ and $P_i$ at each pressure stop were calculated, as their standard deviation, and they are shown in red in the following Figure 2. The linear fit to this average points is also shown.

![Figure 2: Average resistance vs average pressure measurement shown in red. The linear fit is shown in red as well, yielding a slope of $2.07\times10^{-5}$ (MPa)$^{-1}$. The original measurement is shown in black](image)

Finally, each value of and for each red point shown in Figure 2 are calculated, and they are shown in the Table 1. Also the value of is obtained and reported in Table 1, according to Eq. 4.

In Figure 3, the plot of Col. 6 (the piezoresistance term) vs Col. 2, is shown. Also, the linear fit is shown in red.

In Table 2, the results of the three procedures performed are summarized.
Table 1: In Column (Col.) 1 the number of pressure stop is shown. Col. 2 and Col. 3 are the average pressure and resistance at each pressure stop. Col. 4 and Col. 5 are the calculated relative resistance change and pressure change between measurement at values $i$ and $i-1$. Finally, Col. 6 is the piezoresistance term, calculated according to Eq. 4.

<table>
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<tr>
<th>Measurement $i$</th>
<th>Average Pressure $P_i$ (MPa)</th>
<th>Average Resistance $R_i$ (Ω)</th>
<th>$\Delta R = \frac{R_i - R_{i-1}}{R_i}$</th>
<th>$\Delta P_i = P_i - P_{i-1}$ (MPa)</th>
<th>[\frac{\Delta R}{R_i} \times \frac{1}{\Delta P_i} = (\pi_{11} + 2\pi_{12})(\times 10^{-5} \text{MPa}^{-1})]</th>
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</table>

Figure 3: Plot of Col. 6 of Table 1 vs Col. 2 of Table 1. The linear fit is shown in red, yielding a constant term of $2.34 \times 10^{-5} \text{MPa}^{-1}$. 

Our results are compared with the results reported in the literature. In their study, Matsuda et al. reported the values of $\pi_{11}$ and $\pi_{12}$ for $n$-type Si with charge carrier densities of $4 \times 10^{16}$ cm$^{-3}$, $10^{17}$ cm$^{-3}$ and $2 \times 10^{18}$ cm$^{-3}$, and also the values reported by Smith et al. (see Table 3[14]). According to these values, the should have values of $2 \times 10^{-5}$ MPa$^{-1}$, $1 \times 10^{-5}$ MPa$^{-1}$ and $1 \times 10^{-5}$ MPa$^{-1}$ for the $n$-type charge carrier densities of $4 \times 10^{16}$ cm$^{-3}$, $10^{17}$ cm$^{-3}$ and $2 \times 10^{18}$ cm$^{-3}$. Using the values of Smith et al. [15], the value of should be $4 \times 10^{-5}$ MPa$^{-1}$. The deduced values of this study (see Table 2) are in reasonable agreement with this reported value, especially for the case of $4 \times 10^{16}$ cm$^{-3}$, which is very similar to the charge carrier density of our sample.

Finally, the experimental and analytical benefits of the proposed procedure of this study is commented. As it has been shown, a continuous measurement of resistance was done, as pressure was increased. All the measured data were used on the analysis, obtaining the piezoresistance coefficient $\pi_{11} + 2\pi_{12}$ from the slope of the linear fit of the logarithm plot of the measured resistance vs pressure. For comparison, the same measurement was used in two different ways, to compare with the usual procedure found in the literature. In the first one, average resistance and average pressure points were calculated from the measurement, which was deliberately done stopping for some time at certain pressures. This average resistance and average pressures were used: a) as proposed in this article (see second row in Table 2) and b) as usually analysed in the literature (see third row in Table 2). This consumed experimental time and calculation effort, and, as can be seen from column 3 in Table 2, the standard error increased compared with this article's procedure proposal. It can also be seen in Figure 3 that the obtained piezoresistance coefficient $\pi_{11} + 2\pi_{12}$ using the usual procedure done in the literature fluctuates in a wide range of values and several measurements are needed to obtain a reasonable average.

In summary, the procedure proposed here facilitates the obtention of piezoresistance coefficients, as a continuous resistance measurement can be done varying the pressure, and using a simple linear plot of all the measured data, the piezoresistance coefficients can be obtain with more accuracy than the procedure usually done.

Acknowledgements

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### Table 2: Summary of results shown in Figure 1 to 3

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Piezoresistance term $(\pi_{11} + 2\pi_{12})$ (MPa$^{-1}$)</th>
<th>Standard error (MPa$^{-1}$)</th>
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<tr>
<td>This article procedure, using Eq. 6 on all measured data</td>
<td>$2.06 \times 10^{-5}$</td>
<td>$1.171 \times 10^{-7}$</td>
</tr>
<tr>
<td>This article procedure, using Eq. 6 on all averaged measurement points</td>
<td>$2.07 \times 10^{-5}$</td>
<td>$2.834 \times 10^{-7}$</td>
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<td>Usual procedure done in the literature, using Eq. 4, [1-13]</td>
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<td>$5.334 \times 10^{-6}$</td>
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### References


