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The Significance of Limit Cycles in a Neural Model with Poisson and Gauss Connectivity

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Abstract

Isolatedrandomly interconnected nets with chemical markers and noise are investigated, which follow Poisson or Gauss distribution. The obtained results reveal limit cycles. The Poisson limit cycles are primarily large and complex, while the Gauss ones are regularly small.The Poisson limit cycles have various types depending on the shape andtime ofthe transient part, whereas the Gauss ones have the same form, plain, with no particular types of the transient component and are small. The epilepticMEGs follow Poisson distributions with high magnetic amplitudes varying with time and repeatable at time intervals with similar characteristics like the limit cycles. Alternatively the MEGs from healthy subjects have Gauss distributions. The above mentioned differentiations are due to the fact that in Poisson distributed connectivity the activity of the system is organized and synchronized as in epileptic discharges while in Gauss one is random and disordered as in healthy subjects. Limit cycles have been used to form the behavior of many oscillatory systems.

Keywords: Network models, Poisson distribution, Gauss distribution, Limit cycles.

The Neural Net Model

The basic hypotheses of this model have been described in detail previously [1-4].In short, a neural net with N neurotrasmitters (markers)is supposed to be constructed of A neurons. A portion h ($0<h<1$) of them are inhibitory while the rest are excitatory. Each neuron receives on average, μ^+ EPSPs (Excitatory PostSynaptic Potentials) and μ^- IPSPs (Inhibitory PostSynaptic Potentials). K^+ (K^-) is defined the volume of the PSP produced by an excitatory (inhibitory) component. The neurons are also characterized by the absolute refractory period (r) and the synaptic delay τ ($\tau<r<2\tau$). For our theory, r was given the value $r=1$ when refractoriness was assumed, and $r=0$ if not.If a quantity of neurons fire concurrently at time t, then all neural activity resulting from this primary activity will be limited to times $t+\tau$, $t+2\tau$,… If a neuron fires at time t, it produces the PSPs after a synaptic delay τ. PSPs arriving at a neuron are summed at once, and if this sum is greater or equal to θ, then the neuron will fire, or else it will be inactive. If the PSPs are below the threshold (θ) then they will stay with or without decrement for a period called the summation time. The firing is temporary and causes the neuron to be insensible to additional stimulation for the time of a refractory period [5-10].

Poisson distribution

Following the suppositions of previous papers [1-10], the expectation value of the neural activity $<\alpha_{n+1}>$ at $t=(n+1)\tau$, (i.e. the average value) of α_{n+1}generated by a collection of netlets with the same parameters ($\alpha_n, \mu^+, \mu^-, h, K^+, K^-, A, \theta$) at $t=n\tau$ with 2 markers m_a and m_b, is given by:

$$<\alpha_{n+1}>=(1-\alpha_n)\ [m_a \sum_{i=0}^{i_{max}}\sum_{l=0}^{l_{max}} P_l Q_i T_{\delta_a}(\theta_a)+(1-m_a)\sum_{i'=0}^{i'_{max}}\sum_{l'=0}^{l'_{max}} P'_{l'} Q'_{i'} T_{\delta_b}(\theta_b)] \quad (1)$$

where $P_l, Q_i, P`_{l`}, Q`_{i`}$ are the possibilities that the neuron will receive l EPSPs, i IPSPs or $l`$-EPSPs, $i`$-IPSPs, at time $t=(n+1)\tau$ in the subsystems a or b. These probabilities are given by:

$$P_l = \exp(-\alpha_n \mu_a^+ (1-h_a) m_a)(-\alpha_n \mu_a^+ (1-h_a) m_a)^l / l!$$

$$Q_i = \exp(-\alpha_n \mu_a^- h_a m_a)(-\alpha_n \mu_a^- h_a m_a)^i / i!$$

$$P`_{l`} = \exp(-\alpha_n \mu_b^+ (1-h_b)(1-m_a))(-\alpha_n \mu_b^+ (1-h_b)(1-m_a))^{l`} / l`!$$

$$Q`_{i`} = \exp(-\alpha_n \mu_b^- h_b (1-m_a))(-\alpha_n \mu_b^- h_b (1-m_a))^{i`} / i`! \quad (2)$$

The higher limits in the sums in equation (1) are given by:

$$l_{max} = A\ \alpha_n \mu_a^+ (1-h_a)\ m_a$$

$$l_{max`} = A\ \alpha_n \mu_b^+ (1-h_b)(1-m_a)$$

$$i_{max} = A \alpha_n \mu_a^- h_a m_a$$

$$i_{max`} = A\ \alpha_n \mu_b^- h_b (1-m_a) \quad (3)$$

$T_{\delta a}(\theta_a)$ and $T_{\delta b}(\theta_b)$ are defined as the probabilities that the

instantaneous neural thresholds are equal to or less than θ_a and θ_b in subsystems *a* and *b* and are given by:

$$T_{\delta_a}(\theta_a) = \frac{1}{\sqrt{2\pi}} \int_{\frac{\bar{\theta}_a - \theta_a}{\delta_a}}^{\infty} \exp(-\frac{x^2}{2})dx \quad T_{\delta_b}(\theta_b) = \frac{1}{\sqrt{2\pi}} \int_{\frac{\bar{\theta}_b - \theta_b}{\delta_b}}^{\infty} \exp(-\frac{x^2}{2})dx \tag{4}$$

In the general case with *N* chemical markers m_j (j=1,…,N) eq. (1) takes the form:

$$<\alpha_{n+1}> = (1-\alpha_n)[\sum_{j=1}^{N=1} m_j \sum_{i_j=0}^{i_{max\,j}} \sum_{1_j=0}^{/_{max\,j}} P_{1j} Q_{ij} T_{\delta_{jj}}(\theta_j) + (1-\sum_{j=1}^{N-1} m_j) \sum_{i'_N=0}^{i'_{maxN}} \sum_{1'_N=0}^{/'_{maxN}} P'_{1'_N} Q'_{i'_N} T_{\delta_N}(\theta_N)] \tag{5}$$

Gauss distribution

If the numbers l_a, l_b, i_a and i_b are sufficiently large, their distributions may be approximated by Gaussian distributions about their average values.

Consequently, the probability that a neuron with marker *a* or *b* will obtain a definite number of EPSPs or IPSPs that will move the membrane potential closer to or further away from the instantaneous threshold will be given by:

$$P_a(\alpha_n, m_a, \bar{\theta}_a) = \frac{1}{\sqrt{2\pi}} \int_{x_{a,n+1}}^{\infty} \exp(\frac{-x^2}{2})dx \quad x_{a,n+1} = (\bar{\theta}_a - \overline{e_{a,n+1}})/\delta_{a,n+1}$$

and (6)

$$P_b(\alpha_n, m_b, \bar{\theta}_b) = \frac{1}{\sqrt{2\pi}} \int_{x_{b,n+1}}^{\infty} \exp(\frac{-x^2}{2})dx \quad x_{b,n+1} = (\bar{\theta}_b - \overline{e_{b,n+1}})/\delta_{b,n+1}$$

In consequence, the possibilities $T_{\delta a}(\theta`_a)$ and $T_{\delta b}(\theta`_b)$ that the instantaneous threshold of a neuron in subsystems a and b is equal to or less than $\theta`_a$ or $\theta`_b$, will be given by:

$$T_{\delta_a}(\theta'_a) = \frac{1}{\sqrt{2\pi}} \int_{\frac{(\bar{\theta}_a - \theta_a)}{\delta_a}}^{\infty} \exp(\frac{-x^2}{2})dx \quad T_{\delta_b}(\theta'_b) = \frac{1}{\sqrt{2\pi}} \int_{\frac{(\bar{\theta}_b - \theta_b)}{\delta_b}}^{\infty} \exp(\frac{-x^2}{2})dx \tag{7}$$

Accordingly, the firing probabilities $P(\alpha_n,\delta_{n+1},\delta_a)$ and $P`(\alpha_n,\delta_{n+1},\delta_b)$ that a neuron in subpopulations a and b, will receive PSPs exceeding the threshold at time $t=(n+1)\tau$ will be given by:

$$P(\alpha_n, \delta_{n+1}, \delta_a) = P(\alpha_n, m_a, \bar{\theta}_a) \sum_{1=0}^{1_a} \sum_{i=0}^{i_a} T_\delta(\theta'_a) = P(\alpha_n, m_a, \bar{\theta}_a) \sum_{1=0}^{1_a} \sum_{i=0}^{i_a} T_\delta(1K^+ + iK^-)$$

and (8)

$$P'(\alpha_n, \delta_{n+1}, \delta_b) = P(\alpha_n, m_b, \bar{\theta}_b) \sum_{1=0}^{1_b} \sum_{i=0}^{i_b} T_\delta(\theta'_b) = P(\alpha_n, m_b, \bar{\theta}_b) \sum_{1=0}^{1_b} \sum_{i=0}^{i_b} T_\delta(1K^+ + iK^-)$$

The general case for an isolated noisy net with *N* markers $m_1, m_2, ..., m_N$, where m_i is the fraction of neurons with the i^{th} marker at time $t=(n+1)\tau$ is given by:

$$<\alpha_{n+1}> = (1-\alpha_n) \sum_{j=1}^{N} m_j P_j(\alpha_n, m_j, \bar{\theta}_j) T_\delta(\overline{1_j} K^+ + \overline{i_j} K^-) \tag{9}$$

Magnetic field

The description of the magnetic field has been described in detail in our previous work [11,12]. In the general case, where the neural net has *N* chemical markers, it is given by the following equation:

$$B_n = \frac{1}{2}\mu_o\varepsilon_o(\alpha_{n+1} - \alpha_n) A (\sum_{j=1}^{N} \mu_j^+ (1-h_j) m_j - \sum_{j=1}^{N} \mu_j^- h_j m_j) = \frac{1}{2}\mu_o\varepsilon_o\Delta\alpha_n A (\sum_{j=1}^{N} \mu_j^+ (1-h_j) m_j - \sum_{j=1}^{N} \mu_j^- h_j m_j) \tag{10}$$

where the neural action α_{n+1} refers to a Poisson or Gauss distribution of connectivity as given in the prior section. In a two-dimensional phase space, a limit cycle is a closed trajectory in phase space having the property that at least one other trajectory spirals into it either as time approaches infinity or as time approaches negative infinity.

The aim of this study was to compare the limit cycles of the theoretical neural net model with Poisson and Gauss distribution of connectivity.

Methods

Using the equations for the prospect activity α_{n+1} of the net for Poisson and Gauss distribution respectively, we obtained phase diagrams. These diagrams give the steady states of the neural activity α_{ss} ($\alpha_{ss} = \alpha_n = \alpha_{n+1}$) plotted versus the standard deviation δ of the inherent noise of the neural net.

In this study we investigated isolated neural nets with 2-5 chemical markers with Poisson or Gauss distributed connectivity and interior noise and examined their phase diagrams. In order to create a limit cycle we examined the time reliance of the magnetic field B_n, versus $\Delta\alpha_n$.

By constructing a limit cycle there is a part that is called transient and characterizes the time during which the system hasn't reached the periodic activity yet. The limit cycles demonstrate that the system has precise structural parameters and repeated periodical modifications. Based on the other hand on the suppositions that

the time is quantized with the unit quantum time the synaptic delay τ, it is not essential to differentiate the neural activity$\alpha_n$as it is usually done for neural models of continuum time, but instead it is taken the difference $\Delta\alpha_n = \alpha_{n+1} - \alpha_n$ which is included as a term in the description of the magnetic field B_n.

Results

Evaluated the obtained phase diagrams we observed the following:

The Poisson phase diagrams are complicated and have closed and open hysteresis loops while the Gauss ones are simple and have only open hysteresis loops (Figure 1A,B).

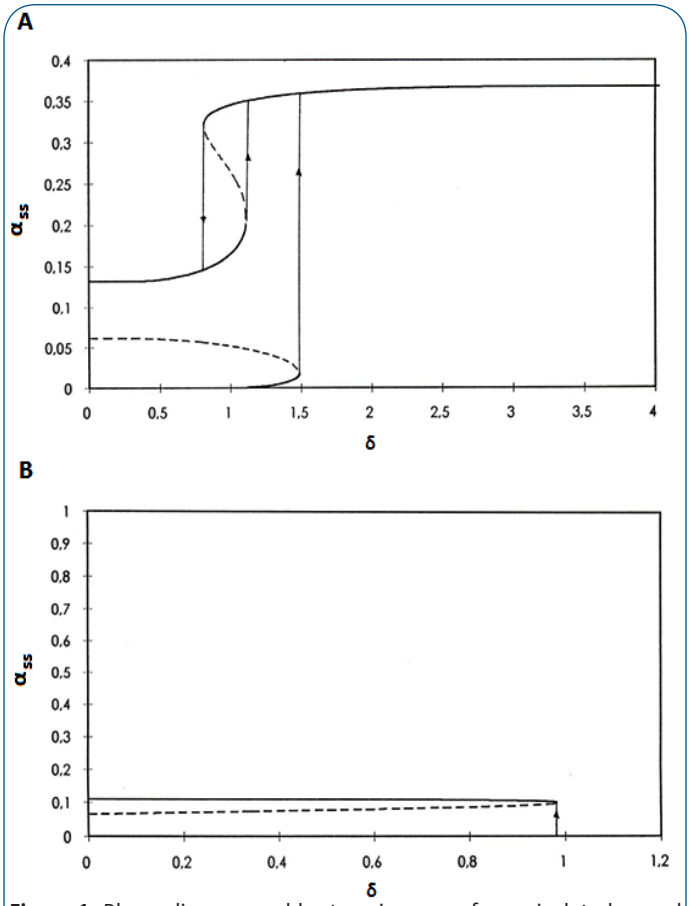

Figure 1: Phase diagram and hysteresis curves for an isolated neural net with noise and 4 chemical markers a,b,c,d. Parameters: m_a=0.7, θ_a=6, $\mu^{\pm}_a$ = 25, h_a=0.015; m_b=0.15, θ_b=6, μ^{+}_b = 23, h_b=0; m_c=0.15, θ_c=4, μ^{+}_c = 370, h_c=0; m_d=0.15, θ_d=4, μ^{+}_d = 370, h_d=0; $K^{\pm}$ = 1, r=1. **A**) Poisson distribution **B**) Gauss distribution

Comparing the obtained limit cycles we observed the following:

The Poisson limited cycles are mostly large and complex, while the Gauss ones are generally small. The transient part in Poisson limited cycles is complex while in Gauss is regularly plain (Figure 2A,B). The Poisson limited cycles have various forms depending on the structure and time of the transient part, while the Gauss ones have the same shape, plain, without particular types of the transient component and are small.

Discussion

The Poisson distributionexpresses the probability of a given number of events occurring in a fixed time interval of space, distance, area or volume, with the assumption that these events occur with a known average rate and independentlyof the time since the last event. Some applications that follow Poisson distribution are: birth defects , genetic mutations, rare diseases, car accidents, traffic flow. If the number of events is very large, then the Gaussian or normal distribution may be used to describe physical events. The Gaussian distribution is a probability distribution that associates the normal random

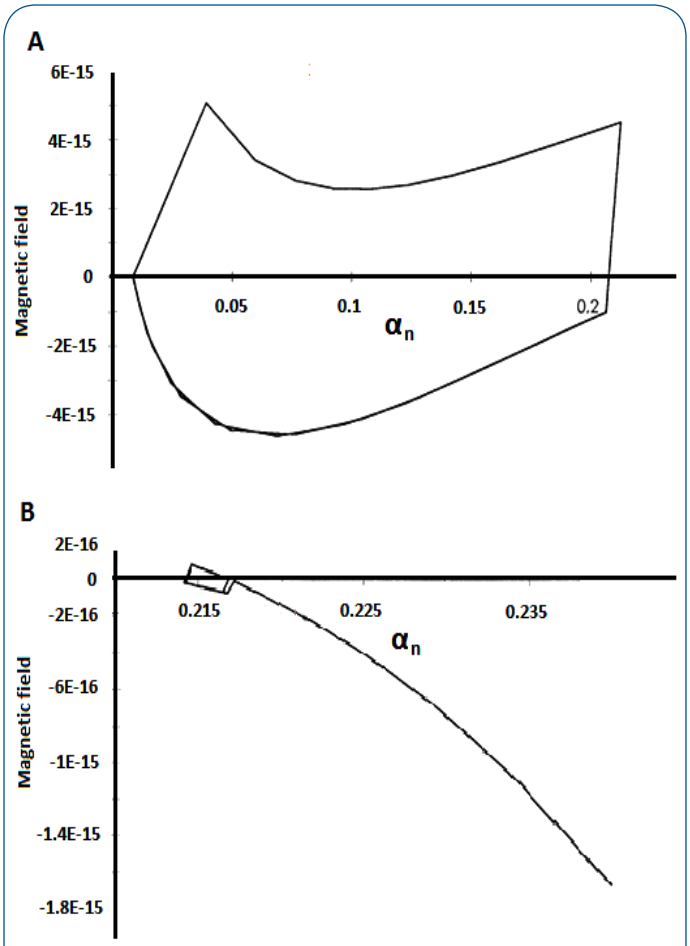

Figure 2: A) Poisson distribution: The limit cycle of an isolated neural net with noise and 2 chemical markers a ,b. Parameters: m_a=0.8, θ_a=4, $\mu^{\pm}_a$ = 9, h_a=0.0301; m_b=0.2, θ_b=4, $\mu^{\pm}_b$ = 115, h_b=0.001; $K^{\pm}$ = 1, r=0. **B**) Gauss distribution: The limit cycle of an isolated neural net with noise and 2 chemical markers a,b. Parameters: m_a=0.7, θ_a=7, $\mu^{\pm}_a$ = 16, h_a=0.0305; m_b=0.3, θ_b=3, $\mu^{\pm}_b$ = 65, h_b=0.015; $K^{\pm}$ = 1, r=0

variable with a cumulative probability. It is an arrangement of a data set in which most values cluster in the middle of the range and the rest taper off symmetrically toward either extreme. The Gaussiandistribution is a very common continuous probability distribution and is often used in the natural and social sciencesto represent real-valued random variables whose distributions are not known.It is important because lots of variables studied in education and psychology are normally distributed,like reading ability, job satisfaction and memory.

In recent years the consequence of structure on function and dynamic behavior in neural nets has been also a topic of considerable attention because the main idea is that this connectivity is given by a binomial distribution. Probabilistic

neural nets were investigated using Poisson or Gauss distributions of inter-neuronal connectivity with the significant conclusion that when a neuron was connected to a relatively small number of units, a Poisson distribution law was proper but if it was connected to a great number of units then a Gauss law was a quite a good estimation. Consequently, Poisson neuronal nets may be viewed as approximately Gauss when the number of synaptic connections is relative large [13-17].

In Figure 1A,B we observe the different hysteresis curves for noisy neural nets with chemical markers with Poisson and Gauss connectivity. As we observe, a small change of δwhich characterizes the spontaneous activity may lead to permanent changes in the steady-state activity of the net. Therefore, the effects of the inherent noise of isolated neural nets are functionally comparable to the effects of sustained inputs to noiseless nets [18]. Another characteristic of the hysteresis loops in both Poisson and Gauss noisy neural nets is that in the Poisson case we have open and closed phase diagrams, while inGauss one we have only open ones. In the case of open phase diagrams, the high state activity is maintained, even with reduced inherent noise of the system, except if we introduce inhibitory inputs. Alternatively, in the case of closed ones, the high stable steady-state activity might return to the lower stable state activity either by lowering δor by introducing inhibitory inputs.

In our former work we compared the hypothetical results with the investigational findings using magnetoencephalographic (MEG) measurements in epileptic patients and healthy volunteers [11,12]. The epilepticMEGs have revealed to follow Poisson distributions with high magnetic amplitudes varying with time and repeatable at time intervals with similar characteristics like the limit cycles. Alternatively the MEGs from normal subjects had Gauss distributions. The epileptic patients had high MEG amplitudes characterized with θ (4-7Hz) or δ (2-3Hz) rhythms and absence of α-rhythm (8-13Hz) whereas the MEG from normal subjects had low amplitudes, higher frequencies and presence of α-rhythm (8-13Hz). The application of transcranial magnetic stimulation (TMS) to epileptic patients changes the distribution of the MEG from Poisson to Gauss. This was in accordance with the connectivity of the theoretical neural model [12,19,20].

Poisson and Gauss distributions have been studied in other models elsewhere. Salinas [21] obtained a mathematical model for solute dynamics assuming that pores follow a Poisson distribution in the lipid phase and that their permeability's follow a Gaussian distribution. He studied a new proposed theory, and suggested a method for finding the mean single pore flux rate from liposome flux assays. Witowski et al. [22]used hidden Markov models to improve the cut-point method in order to achieve a more accurate identification of the sequence of modes of physical activity. The cut-point method is compared with hidden Markov models based on the Poisson distribution, the generalized Poisson distribution and the Gaussian distribution.Seok and Kim [23] studied a fast optimization method for determining the minimizer of the negative Poisson likelihood function for the global analysis of fluorescence lifetime microscopy. Vega and Schnöll-Bitai [24] investigated new approaches for the determination of the extent of symmetric and asymmetric band broadening in size exclusion chromatography using Poisson and Gauss distributions.

In conclusion, due to the fact that the Gauss distribution is a random process, the time course of this system does not exhibit limit cycles or if it does they must be very small, which is in a complete concurrence with the hypothetical neural model. The above mentioned differences are due to the fact that in Poisson connectivity the action of the system is arranged and synchronized like in epileptic discharges and consequently it would achieve limit cycles, while in Gauss connectivity the system is random and disordered as in healthy subjects and the limit cycles are especially small or they don't exist.

Appendix

The subscript *i* is a marker label and indicates the properties of a subpopulation in the netlet characterized by the i^{th} marker.

Parameters

μ^{+}_{i} The average number of neurons receiving excitatory postsynaptic potentials (EPSPs) from one excitatory neuron

μ^{-}_{i} The average number of neurons receiving inhibitory postsynaptic potentials (IPSPs) from one inhibitory neuron

K^{+}_{i}, K^{-}_{i} The size of PSP produced by an excitatory/inhibitory neuron

θ_{i} Firing thresholds of neurons

τ Synaptic delay

A Total number of neurons

h_{i} Fraction of inhibitory neurons

m_{i} Fractions of neurons carrying the i^{th} marker

α_{n} The fractional number of active neurons at time $t=n\tau$

n An integer giving the number of elapsed synaptic delays

δ_{i} Standard deviation of the Gaussian distribution of the neural firing thresholds.

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