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# **Energy Harvesting from Vibrating Piezo-Electric Structures**

N. Ghareeb<sup>\*</sup>, M. Gaith, T. Alloush and M. Al-Sarraf

Australian College of Kuwait

\*Corresponding author: Nader Ghareeb, Department of Mechanical Engineering, Australian College of Kuwait, Kuwait; Tel: +965-1828225 Ext: 4292; Fax: +965 2537 6222; E mail: n.ghareeb@ack.edu.kw

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# Abstract

The process of capturing the energy from a system's environment or surrounding and converting it into usable electrical energy is termed as energy harvesting. One form of energy harvesting is to employ piezoelectric materials to harvest energy from vibrating structures. These materials have the ability to absorb the mechanical energy and transform it into electrical energy that can be used to power other devices.

In this work, it is proposed to theoretically and numerically investigate harvesting energy from mechanical vibrations by a micro electro-mechanical system that is composed of a unimorph cantilevered beam. The relevant equations of vibration, deflection, and natural frequency are derived in order to find the relationship between the tip displacement of the beam and the output voltage across its length. The effect of beam dimension and material properties of the active and inactive layers of the unimorph on the system's performance in terms of output power and vibrational modes frequency is also investigated and presented in different comparison scenarios. Results of the developed model are validated by comparing them to the theoretical and experimental data of similar work done by other researchers, and by using the finite element analysis simulation software ABAQUS'.

 ${\it Keywords:} Piezoelectricity, Deflection, Vibration, Modal analysis.$ 

# Introduction

Piezoelectric energy harvesters are devices that continuously generate electrical power up to 100  $\mu$ W when they are subjected to varying mechanical strain due to vibrations. This generated power from what is considered as autonomous and reliable energy source can be used to drive various sensing and actuating devices [1]. They can be for example implemented in aircraft systems, (e.g. unmanned aircraft and micro air vehicles), or they can be used in wireless sensor networks, indoor-outdoor monitoring, facility management and biomedical applications.

Basic advantages of the piezoelectric energy harvester are its low cost and simplicity to use. It can be easily manufactured and installed because of its small dimensions and large measuring range. Another advantage includes its high mechanical and thermal state capability, so it can withstand high temperatures and different atmospheric pressures. It is also a self-generating device, which means there is no external source of energy required. Moreover, it can withstand great amount of tension and compression. The recent use of aluminum nitride (AlN) as a piezoelectric material has increased fabrication compatibility, enabling the realization of smart integrated systems on chip which include sensors, actuators and energy storage.

Analysis and modeling of piezoelectric energy generators are very important aspects for improved performance and they have been the focus of many recent researches [2-4].

There are usually multiple techniques that are capable of converting the energy produced from system's vibrations into electrical energy. The most prevalent three are electromagnetic, electrostatic, and piezoelectric mechanisms [5]. A very common vibration-based energy harvester consists of a cantilever beam which is generally composed of one or two piezoelectric layers, bounded by electrodes that harvest energy from the beam vibration. The electrodes are externally connected by an electric circuit that is usually modeled as a simple electrical load resistance. A majority of researches had been done on piezoelectric conversion due to the low complexity of its analysis and fabrication, and different beam designs have been considered to maximize the amount of harvested electrical power [6].

However, up to the authors knowledge, none of the previous mentioned researches could show explicitly the relation between the material properties, the geometry of the system, and the resonant frequency and thus on the amount of harvested energy at the same time.

In the present work, an equation incorporating the geometrical and material properties is derived to investigate the static deflection of a piezoelectric energy harvesting structure, in the form of a unimorph beam. The model developed for calculating the deflection is then compared to other analytical and experimental approaches in literature and its accuracy is verified.

These models are taken from the work of Smits [7,8].

The equation of resonant frequency of the piezoelectric unimorph beam is derived by using the effective elastic modulus and density approach, and the results are compared to those from ABAQUS<sup>°</sup>.

The effect of the properties of the active and the non-active layers of the unimorph beam regarding deflection and natural frequency is investigated as well.

### **Theoretical Analysis**

#### Modeling of the static deflection of the cantilevered bimorph

The following derivation for calculating the deflection in terms of applied voltage was done based on the model that was developed using energy density to calculate deflection of piezoelectric cantilever bimorph [7].

The curvature of the beam can be expressed as

$$\delta'' = \frac{3d_{31}V}{2t_p} \tag{1}$$

where  $d_{31}$  is the piezoelectric strain coupling coefficient, V is the voltage, and  $t_{a}$  is the thickness of the piezoelectric layer.

Integrating the above equation with respect to length of the beam (L), yields the slope as

$$\delta = \frac{3d_{31}VL}{2t_p} \tag{2}$$

Integrating once more with respect to (L) yields the deflection of the beam at its end as

$$\delta = \frac{3d_{31}VL^2}{4t_p} \tag{3}$$

The tip deflection of the beam can then be expressed as

$$\delta_t = \delta^{\dagger} \cdot \frac{L^2}{2} \tag{4}$$

The curvature of the bender can now be written as

$$\delta = \frac{2\delta_t}{L^2} = \frac{M}{EI} \tag{5}$$

where *M* is the bending moment and *I* is the moment of inertia. Strain is a linear function of the distance from the neutral axis, expressed as

$$\varepsilon = \frac{M}{EI} y = -\frac{\partial^2 \delta_t}{\partial x^2} \tag{6}$$

where (y) is the distance from the neutral axis as shown in Figure 1. Substituting Eq. (4) in the strain equation yields

$$\varepsilon = \frac{2\delta_t}{L^2} y \tag{7}$$

Using the constitutive piezoelectric equation for strain [10]

$$\varepsilon = sT + d_{31}E \tag{8}$$

with *s* as compliance coefficient, *T* as stress component,  $d_{31}$  as the piezoelectric strain coupling coefficient, and *E* as the electric field component.

Assuming that the stress component (T = 0), an equation describing the relationship between the strain (**e**) and the applied voltage (*V*) can be written as

$$\mathcal{E} = d_{31}E = d_{31}\frac{V}{2t_p} \tag{9}$$

Comparing Eq. (7) and Eq. (9) and solving for the tip deflection yields

$$\delta_t = \frac{d_{31}VL^2}{4t_p y} \tag{10}$$

At any location x from the support of the beam, the deflection will be in the form

$$\delta_t(x,V) = \frac{d_{31}Vx^2}{4t_p y} \tag{11}$$

and the exact value of y will be calculated in the next subsection.

A similar relationship between the deflection of the beam and the applied voltage is derived by [8], which is

$$\delta_t(x, V) = \frac{d_{31}Vx^2}{2t_p(t_{np} + t_p)}$$
(12)

Also Smits et al [7] developed a similar model using the energy density to calculate deflection, which is for this structure

$$\delta_t(x,V) = \frac{3d_{31}Vx^2}{8t_p^2} \tag{13}$$

The predictions of all these models and the experimental results will be compared in Example 1 of the "Results and Discussions" section.

#### Finding the maximum distance from the neutral axis

In this subsection, the location of the neutral axis is calculated. When an electric field is applied, the unimorph will bend and the position of the normal plane  $(t_n)$  is determined by the following equation Li et al [9]

$$\int_{-t_{np}}^{0} E_{np}\left(\frac{z-t_{n}}{r}\right) dz + \int_{0}^{t_{p}} E_{p}\left(\frac{z-t_{n}}{r}\right) dz = 0$$
<sup>(14)</sup>

Where  $t_{np}$  is the thickness of the non-piezoelectric layer, z is the thickness of the unimorph,  $E_{np}$  is the Young's Modulus for the non-piezoelectric layer,  $E_p$  is the Young's Modulus for the piezoelectric layer, r is the radius of curvature, and  $t_n$  is the location of the neutral axis (Figure 1).

The sub-letters (*p*, *np*) refer to the piezoelectric layers and the non-piezoelectric layers, respectively.

Integrating Eq. (14) and solving for  $(t_n)$  yields

$$t_{n} = \frac{1}{2} \cdot \frac{E_{p} t_{p}^{2} - E_{np} t_{np}^{2}}{E_{p} t_{p} + E_{np} t_{np}}$$
(15)



To validate the above solution, it can be assumed that both layers are identical with the same material properties. This will lead to  $t_{x} = 0$ , which means that the neutral axis is exactly in the middle.

Substituting  $t_n$  for y in eq. (11) leads to the final form of the deflection at any location x from the beam support:

$$\delta_t = \frac{1}{2} \frac{(E_p t_p + E_{np} t_{np})}{t_p (E_p t_p^2 - E_{np} t_{np}^2)} d_{31} V x^2$$
(16)

Once the location of the neutral axis is known, the bending modulus per unit length, *D*, can be determined by considering the equilibrium of moments around the neutral axis according to the equation:

$$\int_{-t_{np}}^{0} E_{np} (z - t_n)^2 dz + \int_{0}^{t_p} E_p (z - t_n)^2 dz = 0$$
(17)

Solving the above equation yields the bending modulus per unit length to be

$$D_p = \frac{1}{12} \frac{E_p^2 t_p^4 + E_{np}^2 t_{np}^4 + 2E_p E_{np} t_p t_{np} (2t_p^2 + 2t_{np}^2 + 3t_p t_{np})}{(E_p t_p + E_{np} t_{np})}$$
(18)

#### Estimation of the resonant frequency

The following estimates assume that beams are homogenous, composed of a single uniform material, and of constant cross section. However, equivalent values for Young's modulus and density can be calculated for composite beams by using a weighted average method. The resulting equations describing the resonant frequencies are much more compact, making the scaling analysis far more straightforward. At the beginning, the Euler-Bernoulli beam theory is used to derive the equation of the frequency at the n<sup>th</sup> mode, and then the effective Young's modulus and effective density are calculated and substituted in the frequency equation.

**The Euler-Bernoulli beam theory:** The resonant frequencies of a beam can be estimated using Euler-Bernoulli beam theory [11,12]. It has the form

$$\frac{\partial^4 \delta}{\partial x^4} + \frac{\rho A}{EI} \frac{\partial^2 \delta}{\partial t^2} = 0$$
(19)

where is the beam deflection as a function of position along the beam and time, is the density, *A* is the cross sectional area of the beam, *E* is the Young's modulus, and *I* is the moment of inertia.

For a fixed-free beam, the relevant boundary conditions for a beam of length *L* are:

$$\delta(0, t) = \delta_{\mu}(0, t) = 0$$
 (20)

$$\delta_{rr}(L, t) = \delta_{rrr}(L, t) = 0 \tag{21}$$

The first two boundary conditions indicate that the fixed end of the beam is stationary and the beam is flat at the point of attachment. The other two conditions are at the free end and they indicate that there are no forces or bending moments applied at that point.

Thus, the equation to calculate the frequency of an  $n^{th}$  mode is written as: [13]

$$f_n = \frac{\omega_n}{2\pi} = \frac{(\beta l_n)^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A}}$$
(22)

with () as the n<sup>th</sup> mode eigenvalue.

Substituting I= and A=b.t, the frequency of the beam becomes

$$f_n = \frac{(\beta l)^2}{2\pi} \frac{t}{L^2} \sqrt{\frac{E}{12\rho}}$$
 (23)

where b and t are the width and thickness of the cantilevered unimorph, respectively.

Yi et al [14] wrote the above equation in the form

$$f_n = \frac{(V_n)^2}{2\pi} \cdot \frac{1}{L^2} \cdot \sqrt{\frac{D_p}{m}}$$
(24)

where  $(V_n)^2$  is the dimensionless n<sup>th</sup>-mode eigenvalue and is substituted in the equation instead of  $()^2$ 

**4.3.2 The effective Young's modulus and effective density method:** For a unimorph beam composed of two layers, the mass per unit area is

$$m = \rho_p t_p + \rho_{np} t_{np}$$
(25)

The effect of mass depends on the dimensionless thickness fraction (r), which is used to account for the thickness of each layer in determining the density

$$r_p = \frac{t_p}{t} \quad r_{np} = \frac{t_{np}}{t} \tag{26}$$

This yields the equation expressing the effective density as

$$\rho^{\wedge} = \rho_{\rm p} r_{\rm p} + \rho_{\rm s} r_{\rm s} \tag{27}$$

This means that the mass per unit area becomes

$$m=\rho^{\wedge}.t$$

When introducing the effect of thickness and thickness fraction in Eq. (18), the bending modulus per unit length becomes

$$D_p = \frac{t^3}{12} \frac{E_p^2 r_p^4 + E_{np}^2 r_{np}^4 + 2E_p E_{np} r_p r_{np} (2r_p^2 + 2r_{np}^2 + 3r_p r_{np})}{(E_p r_p + E_{np} r_{np})}$$
(28)

Since the bending modulus per unit length  $(D_p)$  from the original

equations equal to (), it can be said that

$$\hat{E} = \frac{E_p^2 r_p^4 + E_{np}^2 r_{np}^4 + 2E_p E_{np} r_p r_{np} (2r_p^2 + 2r_{np}^2 + 3r_p r_{np})}{(E_p r_p + E_{np} r_{np})}$$
(29)

The bending modulus per unit length is then

$$D_p = \frac{\hat{E}t^3}{12} \tag{30}$$

Thus, Eq. (23) regarding the frequency becomes

$$f_n = \frac{(V_n)^2}{2\pi} \frac{t}{L^2 \sqrt{12}} \sqrt{\frac{\hat{E}}{\hat{\rho}}}$$
(31)

This equation and the equation to represent the mode shapes are programmed in a MATLAB code. They will be validated in Example 2 of the "Results and Discussions" section by performing a modal analysis with the finite element method (FEM) software ABAQUS and comparing the results.

## **Results and Discussions**

#### Example 1: Estimating the static deflection

In this example, the static deflection is estimated for a unimorph beam based on Eq. (16), and the results are compared to the measured and calculated tip deflection by Trownley [8].

The cantilevered unimorph beam used for analysis is approximately long and has a width of The thickness of the active piezo layer and that of the non-active layer. The active layer is made of Aluminum Nitride, and the non-active layer is made of Platinum. The relevant material properties used are shown in Table 1 [8].

**able 1:** Material Properties of the Cantilevered Unimorph Beam Used in Example 1

E <sub>p</sub> (in plane)	292 GPa
$ ho_p$	3200 kg/
<i>d</i> <sub>31</sub>	-1.98 pC/N
E <sub>np</sub>	168 GPa
$\rho_{_{np}}$	21450 kg/

The tip deflection at the end of the beam is depicted in Figure 2. Also the calculated and the measured results of other researchers are shown in the same figure. It can be concluded that they are in good agreement. Comparing equations (14-16), it can be seen that the tip displacement calculated by the authors is a function of the material properties and the geometry as well, while other researchers calculated it only as a function of the geometry in deed. This makes the results from our model more trust-worthy.

To verify the actuation response of the beam, a voltage with different values is applied across the electrodes and the displacement along the longitudinal length of the beam is shown for each voltage and compared to values deduced by other researchers including Smits et al [7]. The results are shown in Figures 3-5 with good agreement.

# Example 2: Estimating the eigenmodes of a cantilevered unimorph beam

In this example, Eq. (31), which was derived to calculate eigenfrequencies (resonant frequencies) and the eigenmodes of the unimorph beam, is used in a self-developed MATLAB code, to obtain the mode shapes. It is validated by comparing the results with those from the FEM software ABAQUS, as mentioned before. The material properties of the beam used in this example are represented in Table 2.

Comparing the first 3 eigenfrequencies, which are depicted in Table 3, shows that the analytical and the numerical results are very close to each other. As seen, there is less than 2% difference between the results for the first three eigenfrequencies.

The first mode of vibration has the lowest resonant frequency and thus provides the most deflection and the most electrical energy. Therefore, energy harvesters are generally designed to operate in the first resonant mode [4]. The mode shapes corresponding to the eigenfrequencies are presented in Figure 6.

In ABAQUS, the beam is modeled as a composite comprising two layers: a non-piezo and a piezo layer. It is easy to create and the simulation time is very short. The beam is constrained from one side while the other side is left free. One of the vibration modes which is produced in ABAQUS is represented in Figure 7.



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TTable 2: Material Properties of the Cantilevered Unimorph Beam Used in Example 2

E <sub>p</sub> (in plane)	66.67 GPa
$ ho_{ ho}$	7800 kg/
E <sub>np</sub>	168 GPa
$\rho_{np}$	7900 kg/

Table 3: The Eigenfrequencies of the Cantilevered Unimorph Beam Used in Example 2

Frequency no.	(Hz)	ABAQUS (Hz)	% Diff
1	7591.13	7708.62	1.52
2	47576.26	48233.26	1.36
3	133262.23	135461.7	1.62

in Table 4, while those for the non-active layer are presented in Table 5.

Material selection is one of the essential considerations in any design, thus, comparisons have been made between different piezoelectric and elastic material combinations to determine which combination yields the highest deflection in terms of voltage. The results of the first three eigenmodes for each combination are represented in Table 6. The results of the static deflection of different active material combinations of the unimorph with steel as non-active material, obtained from numerical solution of the developed model, are shown in Figure 8. Also the deflection over the length of the unimorph beam is shown for different values of voltage in Figures 9-11. It can be seen from these figures that PZT K-500 produces more deflection since it has the highest value of the strain coupling coefficient  $(d_{31})$ .



Figure 7: A mode shape of the unimorph beam produced in ABAQUS

# Example 3: Estimating the deflection of the cantilevered unimorph beam for different material combinations

After the developed equation for beam deflection is validated in Example 1, the effect of the active and non-active material properties is investigated in this example. The cantilevered unimorph beam considered in this example has again a length of 400µm and a width of 100µm. The active (piezoelectric) layer has a thickness of 1µm and the non-active layer has a thickness of 0.2µm. The material properties of the active layers are shown

# Conclusion

In this paper, a new equation to compute the static deflection of a piezoelectric energy harvesting structure in the form of a unimorph beam was derived. Results were compared to other analytical and experimental approaches from literature and have shown good agreement.

Furthermore, the equation of resonant frequency of the piezoelectric unimorph beam using effective mass and effective density was derived and the resonant frequencies were compared

**Table 4:** Material Properties of Active Layers of the CantileveredUnimorph Beam Used in Example 3

Piezoelectric Materials		
Lead Zicronate Titanate K-180		
Young's Modulus (E)	71 GPa	
Density (ρ)	7700 kg/	
Poisson's ratio (v)	0.31	
Strain coupling coefficient ()	-60 pC/N	
Lead Zicronate Titanate K-350		
Young's Modulus (E)	54 GPa	
Density (ρ)	7700 kg/	
Poisson's ratio (v)	0.31	
Strain coupling coefficient ()	-175 pC/N	
Lead Zicronate Titanate K-500		
Young's Modulus (E)	45 GPa	
Density (ρ)	7700 kg/	
Poisson's ratio (v)	0.31	
Strain coupling coefficient ()	-220 pC/N	

 Table 5: Material Properties of Non-active Layers of the Unimorph

 Beam Used in Example 3

Non-active Elastic Materials			
Stainless steel			
Young's Modulus (E)	190 GPa		
Density ( $ ho$ )	4510 kg/m <sup>3</sup>		
Poisson's ratio (v)	0.265		
Platinum			
Young's Modulus (E)	168 GPa		
Density (ρ)	21450 kg/ m <sup>3</sup>		
Poisson's ratio (v)	0.31		
Lithium Niobate			
Young's Modulus (E)	186 GPa		
Density ( $ ho$ )	4650 kg/ m <sup>3</sup>		
Poisson's ratio (v)	0.241		

**Table 6:** Effect of Changing the Material of the Active and Non-active

 Layers on the Frequencies of Beam

	K-180	K-350	K-500
Platinum	f <sub>1</sub> = 3811 Hz	f <sub>1</sub> = 3508 Hz	f <sub>1</sub> = 3317 Hz
	f <sub>2</sub> = 23885 Hz	f <sub>2</sub> = 21989 Hz	f <sub>2</sub> = 20789 Hz
	f <sub>3</sub> = 66901 Hz	f <sub>3</sub> = 61591 Hz	f <sub>3</sub> = 58230 Hz
Stainless Steel	$f_1 = 4611 \text{ Hz}$	f <sub>1</sub> = 4242 Hz	f <sub>1</sub> = 4007 Hz
	f <sub>2</sub> = 28898 Hz	f <sub>2</sub> = 26585 Hz	f <sub>2</sub> = 25113 Hz
	f <sub>3</sub> = 80944 Hz	f <sub>3</sub> = 74466 Hz	f <sub>3</sub> = 70343 Hz
Lithium Niobate	f <sub>1</sub> = 4492 Hz	f <sub>1</sub> = 4135 Hz	f <sub>1</sub> = 3910 Hz
	f <sub>2</sub> = 28153 Hz	f <sub>2</sub> = 25918 Hz	f <sub>2</sub> = 24504 Hz
	f <sub>3</sub> = 78857 Hz	f <sub>3</sub> = 72598 Hz	f <sub>3</sub> = 68636 Hz



to the results from the FEM package ABAQUS and they have shown excellent agreement too.

Finally, the effect of the properties of the active piezoelectric layer and that of the non-active layer of the unimorph beam is investigated regarding its resonant frequency and deflection due to an external voltage. These have demonstrated the significant effect of the material properties on the tip deflection, the resonant frequency, and hence their effect on the amount of power that can be produced.



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