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### Thermodynamic Analysis of MHD Power Cycle

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#### Abstract

A theoretical model to analyse the performance characteristics of a magnetohydrodynamic generator (MHD) power plant is presented in this work. The internal irreversibilities as well as the external irreversibilities are taken into account in the analysis. General expressions for the power and efficiency are obtained when the MHD generator is operating at constant velocity condition. The results of the power output and the thermal efficiency of the MHD power plant are shown graphically.

#### Introduction

Due to the importance of MHD generator power plant in power industry and its competition with gas turbine power plants, MHD power plant is worth studying. In fact MHD power plant is similar to the conventional Brayton cycle except in the latter a gas turbine is used instead of MHD generator. However the source for producing power is completely different.

Magnetohydrodynamic generator (MHD) is a direct convertor of thermal energy and kinetic energy into electrical energy without any intermediate mechanical energy conversion device. The main principle of MHD is based on Faraday's law of electromagnetic induction/ This law states that when a conductor and a magnetic field moves relative to each other, then a voltage is generated which results in electric field across the generator terminals. The conductor is a hot conductive plasma passes through the magnetic field.

In MHD power plant, as the working fluid which is a gaseous conductor passes through the generator in which there is a magnetic field, a current is generated.

MHD generator is characterized by design simplicity, high operating temperatures as well as possibly high efficiency and absence of rotating parts which cause large centrifugal stresses as the case in gas turbines.

Regenerative Brayton cycle has been investigated with internally reversible [1] and internally irreversible conditions [2]. Recent mathematical models for MHD power plant with partial internal

irreversibilities [3] and with variable temperature heat reservoirs [4] have been studied.

The maximum power output of MHD power cycle and the corresponding efficiency without considering the losses in the heat exchanger and in the compressor has been studied [5].

An analysis was carried out to find the maximum power density of MHD power cycle and the corresponding efficiency with considering the loss in the compressor but without considering the loss in the heat exchangers [6].

In the flow systems, thermodynamic irreversibility can be quantified through entropy analysis. Considerable research studies were carried out to examine entropy generation in the flow systems [7]. More research work regarding thermodynamic analysis of MHD an irreversible power plant with different operating conditions and with using regenerator in the power plant can be found in literature [8-11].

The objective of this paper is to investigate the generator effect on the cycle performance for constant velocity operating condition of MHD generator.

#### Analysis

A MHD power plant consists of a compressor, heater, MHD generator and cooler. The T-s diagram of MHD power plant is illustrated in Figure 1. The schematic diagram of MHD generator is shown in Figure 2.

The MHD power plant is running between an infinite heat source at temperature  $T_H$  and infinite heat sink at temperature  $T_L$ . Process 1-2 represents the nonisentropic compression, process 2-3 represents the isobaric heat addition in the hot side heat exchanger from an external heat source, process 3-4 represents the nonisentropic expansion process in the MHD generator, process 4-1 represents the isobaric heat rejection in the cold side heat exchanger to an external heat sink. The isentropic processes 1-2s along the compressor and 3-4s along the MHD generator are also presented in Figure 1.

Considering counter flow heat exchangers, the rates of heat addition and heat rejection are, respectively,

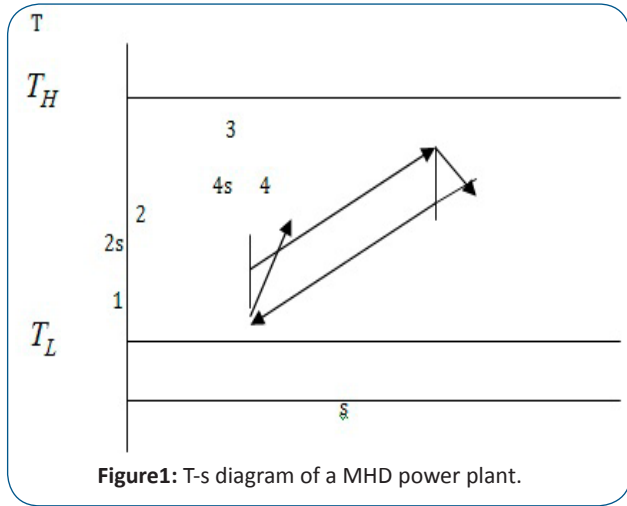


Figure 1: T-s diagram of a MHD power plant.

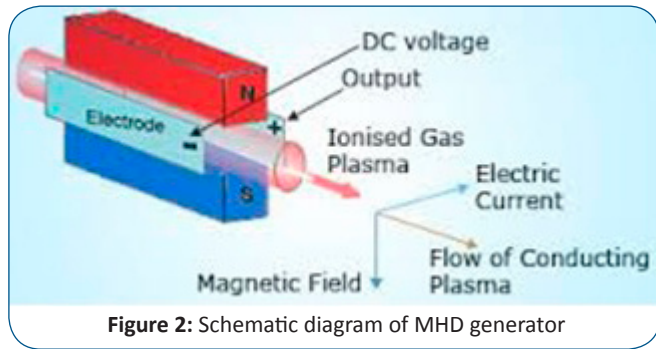


Figure 2: Schematic diagram of MHD generator

$$\text{and } N_L = \frac{U_L A_L}{C_W}$$

The power output and efficiency of MHD power plant are, respectively

$$W = Q_H - Q_L = C_W \epsilon_H (T_H - T_2) - C_W \epsilon_L (T_4 - T_L) \quad (7)$$

$$\eta = \frac{W}{Q_H} \quad (8)$$

Using Equations (1) and (3), and Equations (2) and (4), the following temperatures can be obtained as

$$T_1 = T_4(1 - \epsilon_L) + \epsilon_L T_L \quad (9)$$

$$T_3 = T_2(1 - \epsilon_H) + \epsilon_H T_H \quad (10)$$

$$\text{The compressor efficiency is defined as } \eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \quad (11)$$

For the mathematical modelling of MHD power plant, we have assumed that the working fluid is a compressible ideal gas with constant specific heat, and all cycle processes are internally reversible except for the compressor and MHD generator. Using these assumptions and considering one dimensional steady state flow, the momentum and energy equations of the flowing gas in the MHD generator are written as, respectively,

$$\rho u \frac{du}{dz} + \frac{dp}{dz} + JB = 0 \quad (12)$$

$$\rho u \frac{d}{dz} (h + 0.5u^2) + JE = 0 \quad (13)$$

where  $\rho$  is the gas density,  $u$  gas velocity,  $h$  gas enthalpy,  $p$  gas pressure,  $J$  current density,  $B$  magnetic field and  $E$  electric field.

The electrical efficiency of MHD generator is defined as [13]

$$\eta_e = \frac{JE}{uJB} \quad (14)$$

Equations (11) and (14) give a clear idea about the internal irreversibilities of the compressor and generator, respectively. As the efficiencies of the compressor and generator approach 100 %, the MHD power plant is internally reversible but still externally reversible due to the finite heat transfer between the power plant and its thermal surroundings.

### Constant velocity

In this case the flowing gas in the generator does not slow down but the electrical power comes from the work done by the expansion of the gases against the magnetic forces, i.e. from the enthalpy or thermal energy rather than the kinetic energy of gases.

Using Equations (12) - (14) gives

$$\left( \frac{k-1}{k} \right) \eta_e \frac{dp}{p} = \frac{dT}{T} \quad (15)$$

$$Q_H = \frac{U_H A_H [(T_H - T_2) - (T_H - T_3)]}{\ln \left[ \frac{T_H - T_2}{T_H - T_3} \right]} = C_W (T_3 - T_2) \quad (1)$$

$$Q_L = \frac{U_L A_L [(T_4 - T_L) - (T_1 - T_L)]}{\ln \left[ \frac{T_4 - T_L}{T_1 - T_L} \right]} = C_W (T_4 - T_1) \quad (2)$$

where  $U_H A_H$  is the heat conductance on the hot side heat exchanger,  $U_L A_L$  heat conductance on the cold side heat exchanger (overall heat transfer coefficient -area product) and  $C_W$  capacity rate of the working fluid.

From Equations (1) and (2), we obtain

$$Q_H = C_W \epsilon_H (T_H - T_2) \quad (3)$$

$$Q_L = C_W \epsilon_L (T_4 - T_L) \quad (4)$$

where  $\epsilon_H$  and  $\epsilon_L$  are, respectively, the effectivenesses of the hot side heat exchanger and cold side heat exchanger, for counter flow heat exchangers, they are given by [12].

$$\epsilon_H = 1 - \exp(-N_H) \quad (5)$$

$$\epsilon_L = 1 - \exp(-N_L) \quad (6)$$

The number of heat transfer units  $N_H$  and  $N_L$  are based on

minimum heat capacity rates, which are defined as  $N_H = \frac{U_H A_H}{C_W}$

where  $k$  is the specific heat ratio of the working fluid.

Equation (15) can be integrated for the process 3-4 to give

$$\frac{T_3}{T_4} = X^{\eta_e} \tag{16}$$

where the temperature ratio  $X$  is  $X = \frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$  (17)

By using Equations (7), (9) – (11) and (15) – (17), the power output  $W$  can be written in terms of  $X$  only, and defining a

dimensionless power output as  $W_U^* = \frac{W}{C_w T_L}$ , we obtain

$$W_U^* = \frac{aX^{\eta_e+1} + (b\eta_c - a)X^{\eta_e} + cX - b\eta_c - c}{\eta_c X^{\eta_e} + dX + d(\eta_c - 1)} \tag{18}$$

The efficiency for constant gas velocity case can be expressed as

$$\eta_U = \frac{aX^{\eta_e+1} + (b\eta_c - a)X^{\eta_e} + cX - b\eta_c - c}{aX^{\eta_e+1} + [\eta_c(a+e) - a]X^{\eta_e} + (f-e)X + (f-e)(\eta_c - 1)} \tag{19}$$

where

$$a = -\varepsilon_L \varepsilon_H \tag{20a}$$

$$b = \gamma \varepsilon_H - \varepsilon_L \varepsilon_H + \varepsilon_L \tag{20b}$$

$$c = -\gamma \varepsilon_H (1 - \varepsilon_L) + \varepsilon_L \varepsilon_H - \varepsilon_L \tag{20c}$$

$$d = -(1 - \varepsilon_L)(1 - \varepsilon_H) \tag{20d}$$

$$e = \gamma \varepsilon_H \tag{20e}$$

$$f = \gamma \varepsilon_L \varepsilon_H \tag{20f}$$

where  $\gamma$  is defined as  $\gamma = \frac{T_H}{T_L}$ .

### Efficiency at maximum power

The maximum dimensionless power outputs for constant velocity and constant Mach number can be found by maximizing Equations

$$(18) \text{ with respect to } X \text{ as } \frac{dW_U^*}{dX} = 0 \tag{21}$$

The solution of Equation (21) is too complex to obtain a simple analytical solution for the power output, hence a numerical solution is given in the next section.

For  $\eta_e = \eta_c = 1$  and  $\varepsilon_H = \varepsilon_L$ , Equation (21) gives

$X_{opt} = \gamma^{1/2}$ . For this special case the efficiency of MHD power plant at maximum power output is obtained as

$$\eta = 1 - \left(\frac{T_L}{T_H}\right)^{1/2} \tag{22}$$

which is known as Curzon and Ahlborn efficiency.

## Results and Discussion

In this section, we present the results for constant velocity MHD generator for  $k = 1.4$ ,  $N_H = N_L = 1$  and

$$\gamma = \frac{T_H}{T_L} = 5.$$

The effect of the temperature ratio  $X$  on the dimensionless power output  $W_U^*$  is shown in Figure 3 for different values of MHD generator electrical efficiency  $\eta_e$  and compressor efficiency  $\eta_c$ . The figure shows that there is an optimum value of temperature ratio  $X$  for which  $W_U^*$  is maximum. The figure also shows that as  $\eta_e$  and  $\eta_c$  decrease the power plant produces less power output.

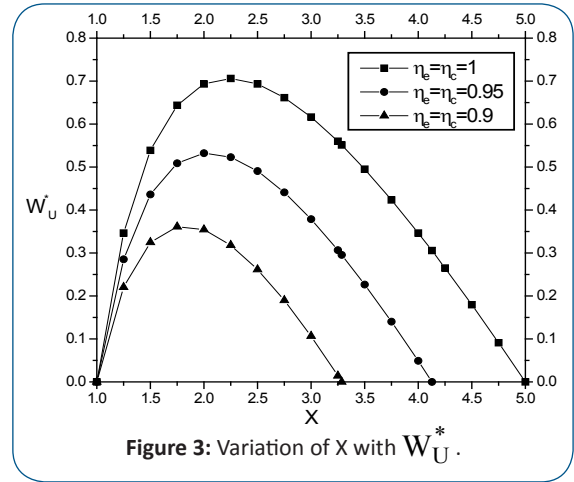


Figure 3: Variation of  $X$  with  $W_U^*$ .

Figure 4 shows the variation of  $X$  with  $\eta_U$ . For  $\eta_e = \eta_c = 1$  (endoreversible cycle), the efficiency of MHD power plant in-

creases monotonically with  $X$ , however for  $\eta_e < 1$  and  $\eta_c < 1$ , there exists a maximum. As can be seen from Figures 3 and 4, the maximum efficiency of MHD power plant occurs at higher  $X$  values than that of the maximum dimensionless power output.

The efficiency of the generator and the efficiency of the compressor are less than one means that there are losses in these two devices and these losses are known as internal irreversibility losses. These losses are diminished when the pump and generator efficiencies are equal to one. This can be seen in Figure 3 where the highest power output is always obtained for  $\eta_e = \eta_c = 1$ , and the power output decreases as these efficiencies go below one.

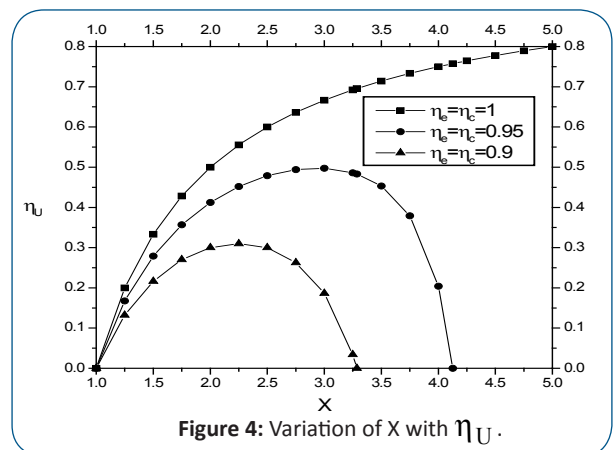


Figure 4: Variation of  $X$  with  $\eta_U$ .

Moreover, in Figure 4, the efficiency of the power plant is always highest for  $\eta_e = \eta_c = 1$  at any value of the compression value. The results of Figures 3 and 4 are presented in terms of  $X$  which contains the pressure ration in the compressor. The figures show that increasing the pressure ratio in the compressor will not always result in a better performance of power plant (power output and efficiency) except for  $\eta_U$  at  $\eta_e = \eta_c = 1$  it is always increasing with pressure ratio.

One of the important points that worth mentioning is for  $\eta_e = \eta_c = 1$ , the working fluid has zero electrical resistivity and the constant velocity case is internally reversible by which it becomes identical to Joule-Brayton cycle. The results show that for  $\eta_e = \eta_c = 1$ , the cycle efficiency reached Carnot efficiency at  $X = 5$  for which the power output is zero, hence the cycle is internally and externally reversible, i.e. the heat engine is running at zero speed however the efficiency at  $X = 1$  is zero and hence zero power output, i.e. the heat engine is running at infinite speed.

## Conclusions

An irreversible MHD power plant operating between an infinite heat source and heat sink has been analysed by using finite-time thermodynamics. The non-isentropic processes in the compressor and generator have been taken into account by introducing the compressor and generator efficiencies. The non-isentropic processes lower the power output as well as efficiency when compared with the endoreversible MHD power plant under the same operating conditions. The model presents a more realistic approach toward analysing MHD power plant by considering the internal irrversibilities as well as the external irrversibilities in the results.

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