VERIZONA PUBLISHER
Publishing Your Ideas

www.verizonaonlinepublishing.com
Vol: 1, Issue: 1

Journal of Robotics and Mechanical Engineering Research

Effect of Heat Exchangers Connection on Effectiveness

Voitto W. Kotiaho[1], Markku J. Lampinen[1] and M. El Haj Assad[*2]

[1]Aalto University, School of Science and Technology, P O Box 14100, FIN-00076, Finland
[2]Australian College of Kuwait, P O Box 1411Safat-13015 Kuwait

***Corresponding author:** Dr. Mamdouh El Haj Assad, Associate Professor/Head of Mechanical Engineering Department, Australian College of Kuwait, Mechanical Engineering Department, PO Box 1411, Safat-13015Kuwait, Tel: +965-1828225 Ext: 4179, Fax: +965-2537-6222; Email: m.assad@ack.edu.kw

Article Type: Research, **Submission Date**: 19 February 2015, **Accepted Date:** 04 May 2015, **Published Date:** 25 May 2015.

Citation: Voitto W. Kotiaho, Markku J. Lampinen and M. El Haj Assad (2015) Effect of Heat Exchangers Connection on Effectiveness. J Robot Mech Eng Resr 1(1): 11-17.

Abstract

In this work, the performance of the recuperative type heat exchangers connected in series is studied. General expressions for the effectiveness of counter flow and parallel flow connections for any type of heat exchangers have been derived. It is proved that the effectiveness of the heat exchanger is only function of the thermal conductance and heat capacity flows. A numerical example is given in order to find how many cross flow heat exchangers connected in series would give the same effectiveness as that of single counterflow heat exchanger.

Introduction

Heat exchangers are devices used to transfer heat between two fluids at different temperatures. The goal of heat exchanger design is to relate the inlet and outlet temperature, the overall heat transfer coefficient, and the geometry of the heat exchanger to the rate of heat transfer between the two fluids.

Heat exchangers are extensively used in power plants as boilers, condensers, feedwater heaters, superheaters, economizers and air heaters; in refrigeration and air-conditioning equipments as evaporators and condensers; and in many other applications. Effect of the maximum and minimum heat capacitance on the performance of heat exchangers from entropy generation point of view has been investigated [1]. Theoretical analysis of counterflow heat exchangers [2] and parallel flow heat exchangers [3] with a heat source within the hot fluid has been studied. Moreover heat exchangers of any type can be connected in series for certain purposes. More details on heat exchanger design and application can be found in literature [4-7].

Heat exchangers can be connected in series in counter flow connection as shown Figure 1a or in parallel flow connection as shown in Figure 1b, while the heat exchangers connected in series in both connections can be any type of heat exchanger.

In order to avoid any misunderstandings between concepts counter-flow unit and counter connection as well as between parallel-flow unit and parallel connection, a counter connection of three parallel flow units (parallel flow heat exchangers) is presented in Figure 2.

In series connection, the overall conductance of the connected units is equal to the sum of the conductances of all individual units.

The main objective of this work is to derive a general expression for the overall effectiveness of the series connection as a function of the effectiveness's of whole units in that connection. Moreover, this study purpose is to obtain the best performance of the heat exchanger based on their connection.

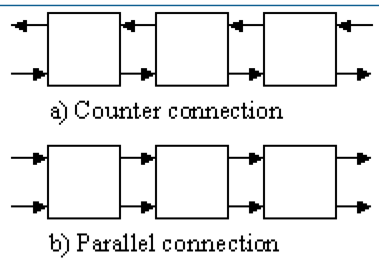

Figure1: Counter- and parallel connection.

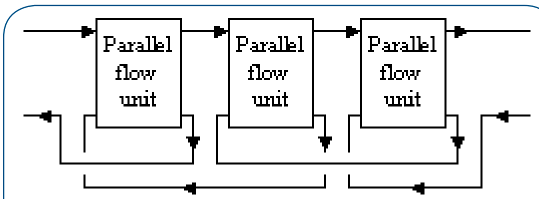

Figure 2: Parallel connection with parallel flow units.

Counter connection

Let us designate by T and $\dot{C}_T$ as the temperature and heat capacity of hot flow; and by t and $\dot{C}_T$ as the temperature and heat capacity of cold flow, respectively.

Consider the counter connection as shown in Figure 3 and assume first that $\dot{C}_T > \dot{C}_t$.

Figure 3 is a counter connection with counter flow heat exchangers (counter units). However units can be any type of heat exchanger since the same mathematical formulation is valid for them; this is because we only consider the temperatures of the hot and cold flows at the inlet and exit of the unit.

Consider unit 1 of Figure 3.

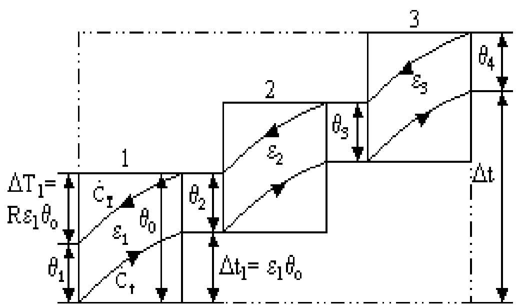

Figure 3: Counter connection with counter flow units.

The heat balance can be written as $\dot{C}_t\Delta t_1 = \dot{C}_T\Delta T_1$ and effectiveness of the unit is $\varepsilon_1 = \Delta t_1/\theta_o$. From these expressions it follows that $\Delta t_1 = \varepsilon_1\theta_o$ and $\Delta T_1 = (\dot{C}_t/\dot{C}_T)\Delta t_1 = R\varepsilon_1\theta_o$, where $R \equiv \dot{C}_{min}/\dot{C}_{max} = \dot{C}_t/\dot{C}_T$. Hence $\theta_2 = \theta_o - \varepsilon_1\theta_o$ and $\theta_1 = \theta_o - R\varepsilon_1\theta_o$, we obtain $\frac{\theta_2}{\theta_1} = \frac{1-\varepsilon_1}{1-R\varepsilon_1}$. Correspondingly for the other units $\frac{\theta_3}{\theta_2} = \frac{1-\varepsilon_2}{1-R\varepsilon_2}$ and $\frac{\theta_4}{\theta_3} = \frac{1-\varepsilon_3}{1-R\varepsilon_3}$. Multiply $\frac{\theta_2}{\theta_1}\cdot\frac{\theta_3}{\theta_2}\cdot\frac{\theta_4}{\theta_3}$ we get $\frac{\theta_4}{\theta_1} = \frac{1-\varepsilon_1}{1-R\varepsilon_1}\frac{1-\varepsilon_2}{1-R\varepsilon_2}\frac{1-\varepsilon_3}{1-R\varepsilon_3}$.

The total heat balance of the connection is $\dot{C}_t\Delta t = \dot{C}_T(\theta_4 + \Delta t - \theta_1)$, where $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$. Hence $R\Delta t = \theta_4 + \Delta t - \theta_1$ and $\Delta t = \frac{\theta_4 - \theta_1}{R-1}$ for $R \neq 1$, and $\theta_4 = \theta_1 = \theta_2 = \theta_3$. for $R = 1$

On the other hand the effectiveness of the whole connection is $\varepsilon = \frac{\Delta t}{\Delta t + \theta_4}$ and thus $1-\varepsilon = \frac{\theta_4}{\Delta t + \theta_4}$. Now let us define an auxiliary variable as

$$\delta \equiv \frac{\varepsilon}{1-\varepsilon} \tag{1}$$

The auxiliary variable is obtained as

$$\delta = \frac{\varepsilon}{1-\varepsilon} = \frac{\Delta t}{\theta_4} = \frac{\theta_4 - \theta_1}{\theta_4(R-1)} = \frac{1-\frac{\theta_1}{\theta_4}}{R-1} = \frac{\frac{\theta_1}{\theta_4}-1}{1-R}$$

Substituting the expression of θ_4/θ_1 in the above equation, we get

$$\delta = \frac{\left(\frac{1-R\varepsilon_1}{1-\varepsilon_1}\frac{1-R\varepsilon_2}{1-\varepsilon_2}\frac{1-R\varepsilon_3}{1-\varepsilon_3}\right)-1}{1-R}$$

or in general form

$$\delta = \frac{1}{1-R}\left[\prod_{k=1}^{n}\left(\frac{1-R\varepsilon_k}{1-\varepsilon_k}\right)-1\right].$$

Then by using Eq. (1), the effectiveness of the whole connection for $0 \le R < 1$ is obtained as

$$\varepsilon = \frac{\prod_{k=1}^{n}\left(\frac{1-R\varepsilon_k}{1-\varepsilon_k}\right)-1}{\prod_{k=1}^{n}\left(\frac{1-R\varepsilon_k}{1-\varepsilon_k}\right)-R} \tag{2}$$

In the same way as above, a general expression for the whole connection (counter connection) effectiveness for $\dot{C}_T < \dot{C}_t$, i.e. $R = \dot{C}_T/\dot{C}_t$, can be obtained, which is the same as that of Eq. (2). So the heat capacitance ratio, R, does not change mathematically the general expression for the effectiveness.

For $R = 1$, $\Delta t_1 = \Delta T_1$, $\Delta t_2 = \Delta T_2$ and $\Delta t_3 = \Delta T_3$, hence we obtain $\varepsilon = \frac{\Delta t_1 + \Delta t_2 + \Delta t_3}{\theta_4 + \Delta t_1 + \Delta t_2 + \Delta t_3}$ and thus we obtain $1-\varepsilon = \frac{\theta_4}{\theta_4 + \Delta t_1 + \Delta t_2 + \Delta t_3}$.

Then the auxiliary variable is obtained as

$$\delta = \frac{\varepsilon}{1-\varepsilon} = \frac{\Delta t_1 + \Delta t_2 + \Delta t_3}{\theta_4} = \frac{\Delta t_1}{\theta_1} + \frac{\Delta t_2}{\theta_2} + \frac{\Delta t_3}{\theta_3},$$

since $\theta_4 = \theta_1 = \theta_2 = \theta_3$ for $R = 1$. On the other hand, the effectiveness of unit 1 is $\varepsilon_1 = \frac{\Delta t_1}{\theta_2 + \Delta t_1} = \frac{\Delta t_1}{\theta_1 + \Delta t_1}$, then $1-\varepsilon_1 = \frac{\theta_1}{\theta_1 + \Delta t_1}$ and $\delta_1 = \frac{\varepsilon_1}{1-\varepsilon_1} = \frac{\Delta t_1}{\theta_1}$. Correspondingly, $\delta_2 = \frac{\varepsilon_2}{1-\varepsilon_2} = \frac{\Delta t_2}{\theta_2}$ and $\delta_3 = \frac{\varepsilon_3}{1-\varepsilon_3} = \frac{\Delta t_3}{\theta_3}$, hence we get $\delta = \delta_1 + \delta_2 + \delta_3$ or generally $\delta = \sum_{k=1}^{n}\delta_k$.

The effectiveness of the whole connection for R = 1 can be written in general form as

$$\varepsilon = \frac{\sum_{k=1}^{n} \frac{\varepsilon_k}{1-\varepsilon_k}}{1+\sum_{k=1}^{n} \frac{\varepsilon_k}{1-\varepsilon_k}} \tag{3}$$

When the units are identical and we assume, that the conductance of a unit does not change if its geometry does not change, it is valid that $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = ... = \varepsilon_u$. That is because it can be proven that the effectiveness ε of a heat exchanger is only function of heat capacity flows and conductance as will be shown later, and the heat capacity flows are the same for all the units. Hence it follows from Eqs. (2) and (3) that

$$\varepsilon = \frac{\left(\frac{1-R\varepsilon_u}{1-\varepsilon_u}\right)^n - 1}{\left(\frac{1-R\varepsilon_u}{1-\varepsilon_u}\right)^n - R} \quad \text{for } 0 \le R < 1 \tag{4}$$

and $$\varepsilon = \frac{n\varepsilon_u}{1+(n-1)\varepsilon_u} \quad \text{for } R = 1 \tag{5}$$

where R is either $\dot{C}_t/\dot{C}_T$ or $\dot{C}_T/\dot{C}_t$ (the minimum heat capacity flow is in the nominator) and n is the number of units. In a similar way it is easy to show that Eqs. (2) and (4) are valid also for R = 0.

Equation (5) is obtained by talking the limit of Equation (4) as R tends to 1 and then using L'Hopitals rule as R tends to 1.

Now we consider the case where $n \to \infty$. The heat balance of an individual unit can be written as $\dot{C}_T \Delta T_u = \dot{C}_t \Delta t_u = G'' A_u \bar{\theta}$, where G" is conductance per unit heat transfer area of the heat exchanger walls, A_u is area of the heat exchangers walls and $\bar{\theta}$ is the average temperature difference between cold and hot fluids. As $n \to \infty$ and the size of the connection still remains finite, the size of the units becomes differentially small and it follows that

$\dot{C}_T dT = \dot{C}_t dt = G'' dA\, \theta$, where θ = T - t. But this is the heat balance of a differential unit of a counter-flow exchanger that means that we end up to equations of the ordinary counter-flow exchanger. Thus, as the number of units is very large, we can consider the whole connection as one counter-flow exchanger.

Parallel connection

Consider the parallel connection where the individual units consist of parallel flow heat exchangers as shown in Figure 4.

Let us first consider the case where $\dot{C}_T > \dot{C}_t$, which i.e. $R = \dot{C}_t/\dot{C}_T$. Considering unit 1, the heat balance is $\dot{C}_t \Delta t_1 = -\dot{C}_T \Delta T_1$ since $\Delta T_1 < 0$. The effectiveness of unit 1 is $\varepsilon_1 = \Delta t_1/\theta_o = \Delta t_1/\theta_1$. From these equations it follows that $\Delta t_1 = \varepsilon_1 \theta_1$ and $\Delta T_1 = -(\dot{C}_t/\dot{C}_T)\Delta t_1 = -R\varepsilon_1\theta_1$.

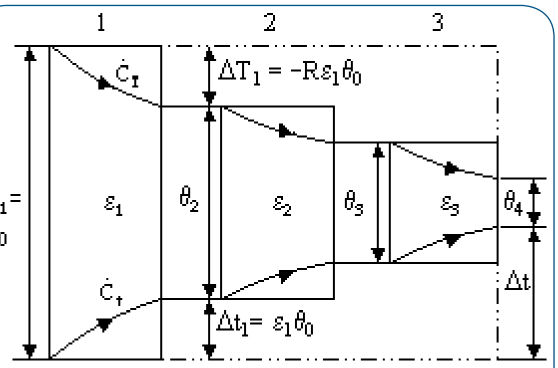

Figure 4: Parallel connection with parallel flow units.

Thus $\theta_2 = \theta_1 - \Delta t_1 - (-\Delta T_1) = \theta_1 - \varepsilon_1\theta_1 - R\varepsilon_1\theta_1 = \theta_1(1 - \varepsilon_1 - R\varepsilon_1)$ which means that

$\frac{\theta_2}{\theta_1} = 1 - \varepsilon_1 - R\varepsilon_1$. Correspondingly for the other units $\frac{\theta_3}{\theta_2} = 1 - \varepsilon_2 - R\varepsilon_2$ and $\frac{\theta_4}{\theta_3} = 1 - \varepsilon_3 - R\varepsilon_3$. Multiplying the temperature difference ratio as $\frac{\theta_2}{\theta_1} \cdot \frac{\theta_3}{\theta_2} \cdot \frac{\theta_4}{\theta_3} \cdot$, the following temperature difference ratio is obtained as

$\frac{\theta_4}{\theta_1} = (1-\varepsilon_1 - R\varepsilon_1)(1-\varepsilon_2 - R\varepsilon_2)(1-\varepsilon_3 - R\varepsilon_3)$ The overall heat balance of the connection is $\dot{C}_t \Delta t = \dot{C}_T(\theta_1 - \Delta t - \theta_4)$ which means that $R\Delta t = \theta_1 - \Delta t - \theta_4$ and $\Delta t = \frac{\theta_1 - \theta_4}{R+1}$.

The effectiveness of the whole connection is

$$\varepsilon = \frac{\Delta t}{\theta_o} = \frac{\Delta t}{\theta_1} = \frac{1}{\theta_1}\frac{\theta_1 - \theta_4}{R+1} = \frac{1 - \frac{\theta_4}{\theta_1}}{R+1}$$
$$= \frac{1-(1-\varepsilon_1 - R\varepsilon_1)(1-\varepsilon_2 - R\varepsilon_2)(1-\varepsilon_3 - R\varepsilon_3)}{R+1}$$

or in a general form

$$\varepsilon = \frac{1-\prod_{k=1}^{n}(1-\varepsilon_k - R\varepsilon_k)}{R+1}. \tag{6}$$

In a similar way we end up with Eq. (6) for the case where $\dot{C}_T < \dot{C}_t$. It is easy to show that Eq. (6) is valid also when R = 1. If all the units of a parallel connection are identical with effectiveness ε_u, it follows from Eq. (6) that

$$\varepsilon = \frac{1-(1-\varepsilon_u - R\varepsilon_u)^n}{R+1}, \tag{7}$$

which is valid for all values of R (when $0 \le R \le 1$).

Let us again consider the case where $n \to \infty$. The heat balance of one unit can be written as $-\dot{C}_T \Delta T_u = \dot{C}_t \Delta t_u = G'' A_u \bar{\theta}$. As $n \to \infty$ and the size of the connection still remains finite, the size of the units is differentially small and it follows that $-\dot{C}_T dT = \dot{C}_t dt = G'' dA\theta$, where $\theta = T - t$. However, this is the heat balance of a differential unit of a parallel-flow exchanger, hence we end up with the equations of the ordinary parallel-flow exchanger. Thus as the number of units is very large, we can consider the whole connection as one parallel-flow exchanger.

A proof that the effectiveness of a heat exchanger is a function of R and NTU only

This proof is based on the Buckingham's Π-theorem and on such a reasonable assumption that the change of temperature of the smaller heat capacity flow in the heat exchanger, ΔT_{max}, is function of four variables which are temperature difference of entering flows θ_o (which is the maximum temperature difference in the whole heat exchanger), minimum heat capacity flow $\dot{C}_{min}$, maximum heat capacity flow $\dot{C}_{max}$, and conductance of the exchanger G. Hence we can write

$\Delta T_{max} = \Delta T_{max}(\theta_o, \dot{C}_{min}, \dot{C}_{max}, G)$.

Mathematically there is a function F that satisfies the following condition $F(\Delta T_{max}, \theta_o, \dot{C}_{min}, \dot{C}_{max}, G) = 0$. The dimensions of the variables are: $[\Delta T_{max}] = [\theta_o] = K$ and $[\dot{C}_{min}] = [\dot{C}_{max}] = [G] = W/K$. We can choose dimensions K and W as basic units, that means there are two linearly independent dimensions in function F. This can be done even though $W = kg\,m^2\,s^{-3}$ is not a basic unit in the standard unit systems, because the linear independence of K and W is all that is demanded. Thus the amount of variables is 5 and the amount of independent dimensions is 2.

According to Buckingham's Π-theorem no information is lost if the function F is derived to a function of dimensionless groups, that is Π-groups, so that the amount of Π-groups in the new function is 3.

We can form, from the variables of function F, for example the following Π-groups: $\Pi_1 = \Delta T_{max}/\theta_o \equiv e$, $\Pi_2 = \dot{C}_{min}/\dot{C}_{max} \equiv R$ and $\Pi_3 = G/\dot{C}_{min} \equiv NTU$. Hence we get a new function f so that $f(\Pi_1, \Pi_2, \Pi_3) = 0$. But this can be derived directly to form $\Pi_1 = \Pi_1(\Pi_2, \Pi_3)$ or $e = e(R, NTU)$, which was to be proved.

Comparison between a counterflow exchanger and a counterflow connection made of crossflow elements.

Considering the three types of heat exchangers, counter-, cross- and parallel flow exchangers, it is well known that the counterflow exchanger has the best effectiveness. Hence, the effectiveness of the counter flow exchanger can be used as a reference. The closer the effectiveness of a connection to the reference one, the better is the connection. An important question is how many elements needed in order that a connection practically performs as counter flow exchanger with the same heat transfer area?

In order to study this, we use the following approximate equation for cross-flow exchangers with both flows unmixed [8]:

$$\varepsilon_{cro} = 1 - \exp\left[\frac{NTU^{0.22}}{R}\left(e^{-R \cdot NTU^{0.78}} - 1\right)\right] \qquad (8)$$

Consider a counter flow exchanger and a counter connection made of cross-flow units so that they both have the same G, $\dot{C}_{min}$ and $\dot{C}_{max}$ and hence the same R and NTU. Let n be the number of the cross flow units. For a cross flow unit

$$NTU_u = NTU/n \qquad (9)$$

Substituting Eq. (9) into Eq (8), we get the effectiveness, ε_u for an individual cross flow unit. Using Eqs. (4) and (5), we get the effectiveness ε_{con} for counter connection.

For a counter-flow exchanger [1], the effectiveness is

$$\varepsilon_{cou} = \begin{cases} 1 - \dfrac{1-R}{\exp[NTU \cdot (1-R)] - R} & \text{for } R < 1 \\ \dfrac{NTU}{1+NTU} & \text{for } R = 1 \end{cases} \qquad (10)$$

Then the percentage difference between both effectivenesses for various values of R, NTU and n can be calculated by $\frac{\varepsilon_{cou} - \varepsilon_{con}}{\varepsilon_{cou}} 100\,\%$.

For example, with values R = 0.75, NTU = 5 and n = 4 we get $e_{cro} = 0.828$ (Eq.(8)), $NTU_u = 1.25$ (Eq.(9)), $e_u = 0.563$ (Eq.(8)), $e_{con} = 0.892$ (Eq.(4)),

$e_{con} = 0.909$ (Eq.(10)) and $\frac{\varepsilon_{cou} - \varepsilon_{con}}{\varepsilon_{cou}} 100\,\% = 2\,\%$.

Results and Discussion

For a counter connection the effectiveness of the connection can be calculated from Eq. (2) or Eq. (3) and especially from Eq. (4) or Eq. (5) when the units are identical.

The corresponding equations for a parallel-flow connection are Eqs. (6) and (7).

The data calculated according to the above procedure is shown graphically in Figures 5a - 5d.

Effectivenesses of counterflow-and crossflow heat exchangers and of counter connections made of crossflow units with different values of R. In each of the figures, the uppermost curve represents the counter flow heat exchanger and the lowermost represents a single cross flow heat exchanger and between the upper- and lowermost curves there are 5 curves, which represent the counter connections made of 2, 4, 6, 8 and 10 with cross flow heat exchanger units. The larger the number of units the closer the curve is to the uppermost curve.

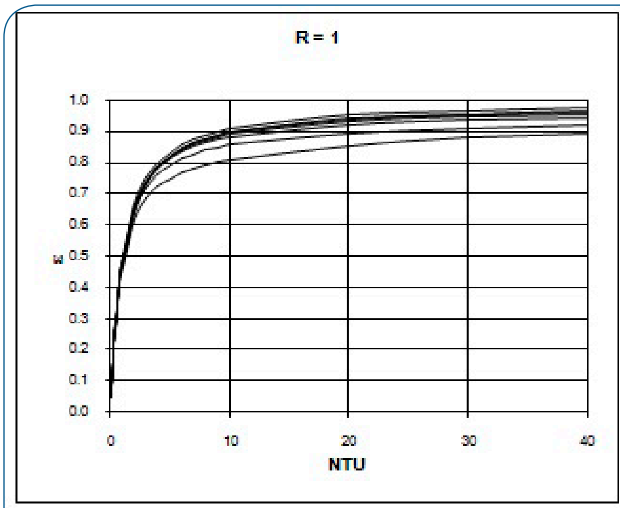

Figure 5a: Variation of ε with NTU.

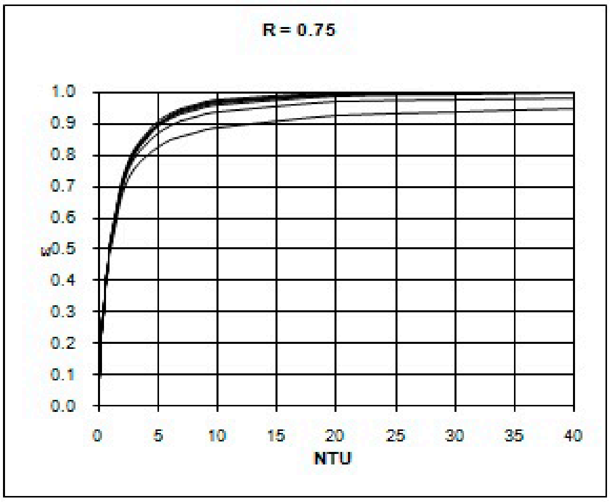

Figure 5b: Variation of ε with NTU.

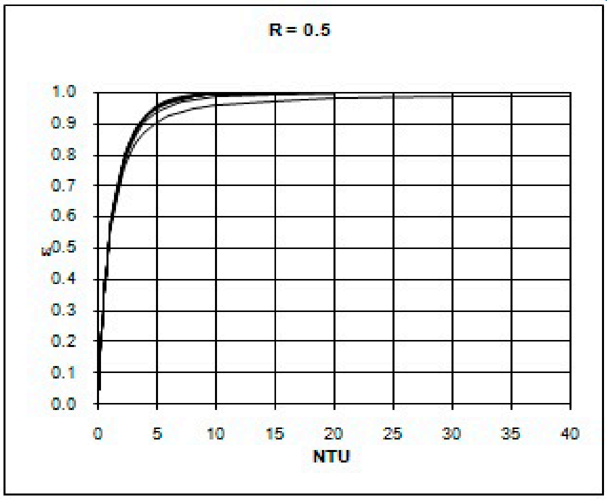

Figure 5c: Variation of ε with NTU.

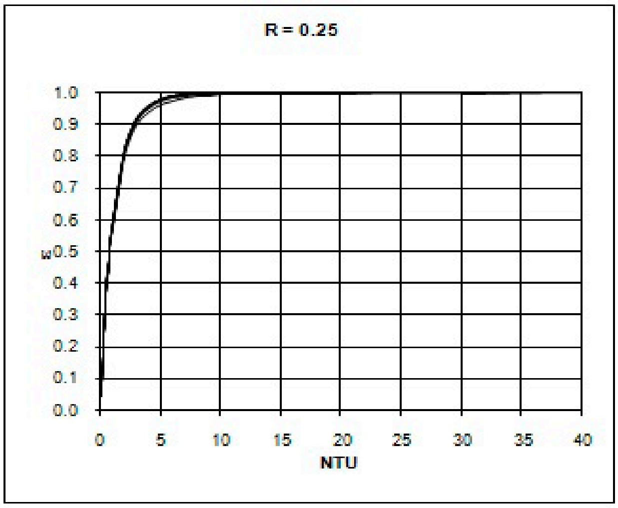

Figure 5d: Variation of ε with NTU.

Figures 5 a - 5 d illustrate that the larger R value the larger the differences of e between individual cases. In case R = 0 which takes place in a phase change, there would be no difference at all and there would be only a single curve in the figure.

Thus we can make a conclusion that the differences are the largest for R = 1, i.e. when the heat capacity flows are the same. This conclusion can be reasoned also in the following way: let us consider a situation where $\dot{C}_{min} < \dot{C}_{max}$. In this case, the temperature change of $\dot{C}_{max}$ is very small hence there is almost constant temperature on the other side of the wall. So it is almost insignificant to which direction $\dot{C}_{min}$ flows compared to $\dot{C}_{max}$. Thus the temperature change of $\dot{C}_{min}$ is less sensitive to the relative flow directions, i.e. the geometry, the larger the difference between $\dot{C}_{min}$ and $\dot{C}_{max}$, i.e. the closer the R is to zero. Hence the temperature change of $\dot{C}_{min}$, and by definition the e is more sensitive to the geometry the closer R value to unity.

Thus, the largest differences between effectivenesses appear in the case R = 1. The largest differences between a single counterflow exchanger and counterflow connections with various number of units of cross flow heat exchangers have been found from the numerical data of Figure 5a. The main results are given in Table 1.

The largest differences of e between a single counterflow exchanger and counterflow connections when R = 1. In the table n = number of units, NTU (max) = the NTU-value where the largest difference occurs, % (max) = largest difference (%), % (NTU = 1) = the difference of e when NTU = 1 (%).

The numerical data was calculated with the accuracy of two decimals and with some values of n there appeared to be two maximum points. In most cases, the maximum point was very close to the point NTU = 1 and for this reason there is an extra row which shows the difference for NTU = 1.

The results indicated by Table 1 can be presented shortly so that two units are needed in order that the difference in all conditions is less than 10 %, for 3 units less than 5 %, for 8 units less than 2 % and for 28 units less than 1 %. But it must be remembered that we considered here the limit case R = 1 and with smaller values of R usually smaller amount of units is needed. Table 1 indicates also that the difference of e between a counterflow and a single crossflow exchanger is never larger than 11%.

n	1	2	3	4	5	6	7	8	9	10
NTU(max)	10.0	17.9	25.6	33.1	40.7	1.1	1.1	1.1	1.1	1.2
%(max)	11.0	6.1	4.2	3.2	2.6	2.3	2.1	2.0	2.0	1.8
%(NTU=1)	6.3	4.1	3.3	2.5	2.5	2.3	2.1	2.0	2.0	1.8
n	11	12	13	14	15	16	17	18	19	20
NTU(max)	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3
%(max)	1.7	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.3	1.2
%(NTU=1)	1.7	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2
n	21	22	23	24	25	26	27	28	29	30
NTU(max)	1.3	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4
%(max)	1.2	1.2	1.2	1.1	1.1	1.0	1.0	1.0	1.0	1.0
%(NTU=1)	1.2	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	0.9

Table 1: Main Results

It can be proven by the dimensional analysis that for any heat exchanger the effectiveness is a function of NTU and R only and does not depend on the incoming temperatures.

Practically not so many crossflow units are needed in order that the effectiveness of a counterflow connection is practically not worse than that of a single counterflow exchanger. It was proved that 8 units are needed in order to have only 2 % difference in effectiveness.

Conclusions

A theoretical formulation was presented in order to obtain the effectiveness of heat exchangers connected in series. The effectiveness was derived for counter and parallel flow heat exchangers connected in series. If the heat exchangers are connected in series they should be connected in counter flow connection in order to achieve the best possible effectiveness.

It was shown that the effectiveness of the heat exchanger is only function of the heat capacitance ratio of the two fluids and the thermal conductance.

Usually only a few units are needed in order that the effectivenes is almost the same as a single counterflow exchanger.

Nomenclature

A area of heat transfer wall, m^2

c_p specific heat, J/kgk

$\dot{C}$ heat capacity rate, W/K, ($\dot{C} = \dot{m} c_p$)

G conductance, W/K

$\dot{m}$ mass flow, kg/s

n number of units

NTU number of heat transfer units, ($NTU = G/\dot{C}_{min}$)

R ratio of heat capacitance, ($R = \dot{C}_{min}/\dot{C}_{max}$)

t cold fluid temperature, K

T hot fluid temperature, K

Greek Letters

δ auxiliary variable

ε effectiveness of a heat exchanger

θ temperature difference, K

Subscripts

cro crossflow exchanger

cou counterflow exchanger

con counterflow connection made of crossflow units

Min minimum

max maximum

t cold fluid

T hot fluid

u unit

References

1. M El Haj Assad. Effect of maximum and minimum heat capacity rate on entropy generation in a heat exchanger. Int. J. Energy Research. 2010; 34(14):1302-1308.doi: 10.1002/er.1674.

2. M El Haj Assad, Voitto Kotiaho. Thermal analysis of a counterflow heat exchanger with a heat source. Int. J. Ambient Energy. 2010; 31(4):211-217.doi:10.1080/01430750.2010.9675814.

3. M El Haj Assad, Voitto Kotiaho. Analysis of a parallel flow heat exchanger with a heat source. Heat Transfer Engineering. 2011; 32(5):384-389. doi:10.1080/01457632.2010.483863.

4. A Bejan. Heat Transfer. New York: Wiley; 1993. 675 p.

5. S Kakac, editor. Boilers, evaporators and condensers. New York: Wiley; 1991. 835 p.

6. WM Kays, AL London. Compact Heat Exchangers. New York: McGraw-Hill; 1984.

7. RK Shah, DP Sekulic. Fundamentals of Heat Exchanger Design. New York: Wiley; 2003. 941 p.

8. FP Incropera, DP DeWitt. Fundamentals of Heat and Mass Transfer. New York: John Wiley &Sons; 1990.